MATHEMATICS I: ALGEBRA/GEOMETRY/STATISTICS

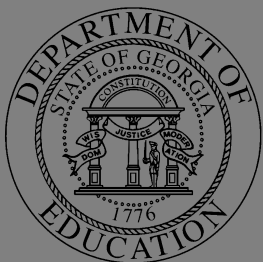# Study

Guide
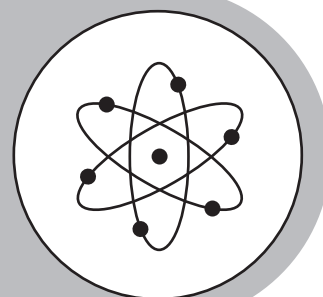
Georgia End-Of-Course Tests

TABLE OF CONTENTS

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INTRODUCTION

This study guide is designed to help students prepare to take the Georgia End-of-Course Test (EOCT) for *Mathematics I*. This study guide provides information about the EOCT, tips on how to prepare for it, and some suggested strategies students can use to perform their best.

What is the EOCT? The EOCT program was created to improve student achievement through effective instruction and assessment of the standards in the Georgia Performance Standards (GPS) specific to the eight EOCT core high school courses. The EOCT program also helps to ensure that all Georgia students have access to a rigorous curriculum that meets high performance standards. The purpose of the EOCT is to provide diagnostic data that can be used to enhance the effectiveness of schools' instructional programs.

The Georgia End-of-Course Testing program is a result of the A+ Educational Reform Act of 2000, O.C.G.A. §20-2-281. This act requires that the Georgia Department of Education create end-of-course assessments for students in grades 9 through 12 for the following core high school subjects:

Mathematics

- Mathematics I: Algebra/Geometry/Statistics
- Mathematics II: Geometry/Algebra II/Statistics

Social Studies

- United States History
- Economics/Business/Free Enterprise

Science

- Biology
- Physical Science

English Language Arts

- Ninth Grade Literature and Composition
- American Literature and Composition

Since the EOCT program is designed to measure a student's mastery of a specific curriculum, if the curriculum changes, so too must the test. This occurred in 2005, when the Georgia Department of Education modified and revised the state's Mathematics curriculum. This new curriculum was implemented in high school classrooms starting in 2007–2008. Due to this change, the *Mathematics I* **EOCT** was developed in order to reflect the updated standards, and this study guide has also been revised appropriately to cover the new set of standards.

HOW TO USE THE STUDY GUIDE

This study guide is designed to help you prepare to take the *Mathematics I* **EOCT**. It will give you valuable information about the EOCT, explain how to prepare to take the EOCT, and provide some opportunities to practice for the EOCT. The study guide is organized into three sections. Each section focuses on a different aspect of the EOCT.

The OVERVIEW OF THE EOCT section on page 4 gives information about the test: dates, time, question format, and number of questions that will be on the *Mathematics I* **EOCT**. This information can help you better understand the testing situation and what you will be asked to do.

The PREPARING FOR THE EOCT section that begins on page 5 provides helpful information on study skills and general test-taking skills and strategies. It explains how to prepare before taking the test and what to do during the test to ensure the best test-taking situation possible.

The TEST CONTENT section that begins on page 11 explains what the *Mathematics I* **EOCT** specifically measures. When you know the test content and how you will be asked to demonstrate your knowledge, it will help you be better prepared for the EOCT. This section also contains some sample EOCT test questions, helpful for gaining an understanding of how a standard may be tested.

With some time, determination, and guided preparation, you will be better prepared to take the *Mathematics I* **EOCT**.

GET IT TOGETHER

In order to make the most of this study guide, you should have the following:

Materials:
* This study guide
* Pen or pencil
* Highlighter
* Paper

Resources:
* Classroom notes
* Mathematics textbook
* A teacher or other adult

Study Space:
* Comfortable (but not too comfortable)
* Good lighting
* Minimal distractions
* Enough work space

Time Commitment:
* When are you going to study?
* How long are you going to study?

Determination:
* Willingness to improve
* Plan for meeting goals

SUGGESTED STEPS FOR USING THIS STUDY GUIDE

1 Familiarize yourself with the structure and purpose of the study guide.
(You should have already read the INTRODUCTION and HOW TO USE THE STUDY GUIDE. Take a few minutes to look through the rest of the study guide to become familiar with how it is arranged.)

2 Learn about the test and expectations of performance.
(Read OVERVIEW OF THE EOCT.)

3 Improve your study skills and test-taking strategies.
(Read PREPARING FOR THE EOCT.)

4 Learn what the test will assess by studying each unit and the strategies for answering questions that assess the standards in the unit.
(Read TEST CONTENT.)

5 Answer the sample test question at the end of each lesson. Check your answer against the answer given to see how well you did.
(See TEST CONTENT.)

OVERVIEW OF THE EOCT

Good test takers understand the importance of knowing as much about a test as possible. This information can help you determine how to study and prepare for the EOCT and how to pace yourself during the test. The box below gives you a snapshot of the *Mathematics I* **EOCT** and other important information.

THE EOCT AT A GLANCE

Administration Dates:

The EOCT has three primary annual testing dates: once in the spring, once in the summer, and once in the winter. There are also mid-month, online tests given in August, September, October, November, February, and March.

Administration Time:

Each EOCT is composed of two sections, and students are given 60 minutes to complete each section. There is also a short stretch break between the two sections of the test.

Question Format:

All the questions on the EOCT are multiple-choice.

Number of Questions:

Each section of the *Mathematics I* **EOCT** contains 31 questions; there are a total of 62 questions on the *Mathematics I* **EOCT**.

Impact on Course Grade:

A student's EOCT score is averaged in as 15% of his/her final course grade.

If you have additional administrative questions regarding the EOCT, please visit the Georgia Department of Education Web site at www.doe.k12.ga.us, see your teacher, or see your school test coordinator.

PREPARING FOR THE EOCT

WARNING!

You cannot prepare for this kind of test in one night. Questions will ask you to apply your knowledge, not list specific facts. Preparing for the EOCT will take time, effort, and practice.

To do your best on the *Mathematics I* **EOCT**, it is important that you take the time necessary to prepare for this test and develop those skills that will help you take the EOCT.

First, you need to make the most of your classroom experiences and test preparation time by using good **study skills**. Second, it is helpful to know general **test-taking strategies** to ensure that you will achieve your best score.

Study Skills

A LOOK AT YOUR STUDY SKILLS

Before you begin preparing for this test, you might want to consider your answers to the following questions. You may write your answers here or on a separate piece of paper.

1. How would you describe yourself as a student?
   Response: _____

2. What are your study skills strengths and/or weaknesses as a student?
   Response: _____

3. How do you typically prepare for a mathematics test?
   Response: _____

4. Are there study methods you find particularly helpful? If so, what are they?
   Response: _____

5. Describe an ideal study situation (environment).
   Response: _____

6. Describe your actual study environment.
   Response: _____

7. What can you change about the way you study to make your study time more productive?
   Response: _____

Effective study skills for preparing for the EOCT can be divided into three categories:

- **Time Management**
- **Organization**
- **Active Participation**

Time Management

Do you have a plan for preparing for the EOCT? Often students have good intentions for studying and preparing for a test, but without a plan, many students fall short of their goals. Here are some strategies to consider when developing your study plan:

- Set realistic goals for what you want to accomplish during each study session and chart your progress.
- Study during your most productive time of the day.
- Study for reasonable amounts of time. Marathon studying is not productive.
- Take frequent breaks. Breaks can help you stay focused. Doing some quick exercises (e.g., sit-ups or jumping jacks) can help you stay alert.
- Be consistent. Establish your routine and stick to it.
- Study the most challenging test content first.
- For each study session, build in time to review what you learned in your last study session.
- Evaluate your accomplishments at the end of each study session.
- Reward yourself for a job well done.

Organization

You don't want to waste your study time. Searching for materials, trying to find a place to study, and debating what and how to study can all keep you from having a productive study session. Get organized and be prepared. Here are a few organizational strategies to consider:

- Establish a study area that has minimal distractions.
- Gather your materials in advance.
- Develop and implement your study plan (see Appendices A–D for sample study plan sheets).

Active Participation

Students who actively study will learn and retain information longer. Active studying also helps you stay more alert and be more productive while learning new information. What is active studying? It can be anything that gets you to interact with the material you are studying. Here are a few suggestions:

♦ Carefully read the information and then DO something with it. Mark the important points with a highlighter, circle it with a pen, write notes on it, or summarize the information in your own words.

♦ Ask questions. As you study, questions often come into your mind. Write them down and actively seek the answers.

♦ Create sample test questions and answer them.

♦ Find a friend who is also planning to take the test and quiz each other.

Test-taking Strategies

There are many test-taking strategies that you can use before and during a test to help you have the most successful testing situation possible. Below are a few questions to help you take a look at your test-taking skills.

A LOOK AT YOUR TEST-TAKING SKILLS

As you prepare to take the EOCT, you might want to consider your answers to the following questions. You may write your answers here or on your own paper.

1. How would you describe your test-taking skills?
   Response: _____

2. How do you feel when you are taking a test?
   Response: _____

3. List the strategies that you already know and use when you are taking a test.
   Response: _____

4. List test-taking behaviors you use that contribute to your success when preparing for and taking a test.
   Response: _____

5. What would you like to learn about taking tests?
   Response: _____

Suggested Strategies to Prepare for the EOCT

Learn from the past. Think about your daily/weekly grades in your mathematics classes (past and present) to answer the following questions:

- In which specific areas of mathematics were you or are you successful?

  Response: _____

- Is there anything that has kept you from achieving higher scores?

  Response: _____

- What changes should you implement to achieve higher scores?

  Response: _____

Before taking the EOCT, work toward removing or minimizing any obstacles that might stand in the way of performing your best. The test preparation ideas and test-taking strategies in this section are designed to help guide you to accomplish this.

Be prepared. The best way to perform well on the EOCT is to be prepared. In order to do this, it is important that you know what standards/skills will be measured on the *Mathematics I EOCT* and then practice understanding and using those standards/skills. The TEST CONTENT section of this study guide is designed to help you understand the specific standards that are on the *Mathematics I EOCT* and give you suggestions for how to study the standards that will be assessed. Take the time to read through this material and follow the study suggestions. You can also ask your math teacher for any suggestions he or she might offer on preparing for the EOCT.

Start now. Don't wait until the last minute to start preparing. Begin early and pace yourself. By preparing a little bit each day, you will retain the information longer and increase your confidence level. Find out when the EOCT will be administered, so you can allocate your time appropriately.

Suggested Strategies the Day before the EOCT
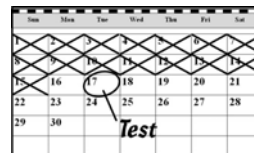
✓ **Review what you learned from this study guide**.

1. Review the general test-taking strategies discussed in the TOP 10 SUGGESTED STRATEGIES DURING THE EOCT on page 10.

2. Review the content information discussed in the TEST CONTENT section beginning on page 11.

3. Focus your attention on the main topic, or topics, that you are most in need of improving.

✓ **Take care of yourself**.

1. Try to get a good night's sleep. Most people need an average of eight hours, but everyone's sleep needs are different.

2. Don't drastically alter your routine. If you go to bed too early, you might lie in bed thinking about the test. You want to get enough sleep so you can do your best.

Suggested Strategies the Morning of the EOCT

Eat a good breakfast. Choose foods high in protein for breakfast (and for lunch if the test is given in the afternoon). Some examples of foods high in protein are peanut butter, meat, and eggs. Protein gives you long-lasting, consistent energy that will stay with you through the test to help you concentrate better. Avoid foods high in sugar content. It is a misconception that sugar sustains energy—after an initial boost, sugar will quickly make you more tired and drained. Also, don't eat too much. A heavy meal can make you feel tired. So think about what you eat before the test.

Dress appropriately. If you are too hot or too cold during the test, it can affect your performance. It is a good idea to dress in layers, so you can stay comfortable, regardless of the room temperature, and keep your mind on the EOCT.

Arrive for the test on time. Racing late into the testing room can cause you to start the test feeling anxious. You want to be on time and prepared.

TOP 10
Suggested Strategies during the EOCT

These general test-taking strategies can help you do your best during the EOCT.

1 **Focus on the test.** Try to block out whatever is going on around you. Take your time and think about what you are asked to do. Listen carefully to all the directions.

2 **Budget your time.** Be sure that you allocate an appropriate amount of time to work on each question on the test.

3 **Take a quick break if you begin to feel tired.** To do this, put your pencil down, relax in your chair, and take a few deep breaths. Then, sit up straight, pick up your pencil, and begin to concentrate on the test again. Remember that each test section is only 60 minutes.

4 **Use positive self-talk.** If you find yourself saying negative things to yourself such as "I can't pass this test," it is important to recognize that you are doing this. Stop and think positive thoughts such as "I prepared for this test, and I am going to do my best." Letting the negative thoughts take over can affect how you take the test and your test score.

5 **Mark in your test booklet.** Mark key ideas or things you want to come back to in your test booklet. Remember that only the answers marked on your answer sheet will be scored.

6 **Read the entire question and the possible answer choices.** It is important to read the entire question so you know what it is asking. Read each possible answer choice. Do not mark the first one that "looks good."

7 **Use what you know.** Draw on what you have learned in class, from this study guide, and during your study sessions to help you answer the questions.

8 **Use content domain-specific strategies to answer the questions.** In the TEST CONTENT section, there are a number of specific strategies that you can use to help improve your test performance. Spend time learning these helpful strategies, so you can use them while taking the test.

9 **Think logically.** If you have tried your best to answer a question but you just aren't sure, use the process of elimination. Look at each possible answer choice. If it doesn't seem like a logical response, eliminate it. Do this until you've narrowed down your choices. If this doesn't work, take your best educated guess. It is better to mark something down than to leave it blank.

10 **Check your answers.** When you have finished the test, go back and check your work.

A WORD ON TEST ANXIETY

It is normal to have some stress when preparing for and taking a test. It is what helps motivate us to study and try our best. Some students, however, experience anxiety that goes beyond normal test "jitters." If you feel you are suffering from test anxiety that is keeping you from performing at your best, please speak to your school counselor, who can direct you to resources to help you address this problem.

TEST CONTENT

Up to this point in this study guide, you have been learning various strategies on how to prepare for and take the EOCT. This section focuses on what will be tested. It also includes sample questions that will let you apply what you have learned in your classes and from this study guide.

This section of the study guide will help you learn and review the various mathematical concepts that will appear on the *Mathematics I* **EOCT**. Since *mathematics* is a broad term that covers many different topics, the state of Georgia has divided it into three major areas of knowledge called **content strands**. The content strands are broad categories. Each of the content strands is broken down into big ideas. These big ideas are called **content standards** or just standards. Each content strand contains standards that cover different ideas related to the content strand. Each question on the EOCT measures an individual standard within a content strand.

The three content strands for the *Mathematics I* **EOCT** are Algebra, Geometry, and Data Analysis and Probability. They are important for several reasons. Together, they cover the major skills and concepts needed to understand and solve mathematical problems. These skills have many practical applications in the real world. Another more immediate reason that the content strands are important has to do with test preparation. The best way to prepare for any test is to study and know the material measured on the test.

This study guide is organized in six **units** that review the material covered within the six units of the Mathematics I GPS Frameworks. It is presented by topic rather than by strand or standard (although those are listed at the beginning of each unit and are integral to each topic). The more you understand about the topics in each unit, the greater your chances of getting a good score on the EOCT.

Studying the Content Standards and Topics
(Unit 1—Unit 6)

You should be familiar with many of the content standards and topics that follow. It makes sense to spend more time studying the content standards and topics that you think may cause you problems. Even so, do not skip over any of them. The TEST CONTENT section has been organized into six units. Each unit is organized by the following features:

- **Introduction:** an overview of what will be discussed in the unit

- **Key Standards:** information about the specific standards that will be addressed

  (NOTE: The names of the standards may not be the exact names used by the Georgia Department of Education.)

- **Main Topics:** the broad subjects covered in the unit

  Each Main Topic includes:
  - **Key Ideas:** definitions of important words and ideas as well as descriptions, examples, and steps for solving problems

  - **Review Examples:** problems with worked-out solutions showing possible ways to answer given questions

  - **EOCT Prep Test Items:** sample multiple-choice questions similar to test items on the *Mathematics I* **EOCT** with answers provided

With some time, determination, and guided preparation, you will be better prepared to take the *Mathematics I* **EOCT**.

Unit 1: Function Families

This unit explores properties of basic quadratic, cubic, absolute value, square root, and rational functions and new language and notation for talking about functions. The discussion of functional characteristics leads to the development of the language of mathematical reasoning, which includes formal discussion of the logical relationships among an implication and its converse, its inverse, and its contrapositive. The discussion of sequences focuses primarily on sequences of numbers and often-used geometric figures and diagrams as illustrations and contexts for investigating various number sequences.

KEY STANDARDS

MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables and simple algebraic techniques.

     a. Represent functions using function notation.

     b. Graph the basic functions $f(x) = x^n$ where $n = 1$ to 3, $f(x) = \sqrt{x}$, $f(x) = |x|$, and

$$f(x) = \frac{1}{x}.$$

     c. Graph transformations of basic functions, including vertical shifts, stretches, and shrinks as well as reflections across the x- and y-axes.

     d. Investigate and explain the characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.

     e. Relate to a given context the characteristics of a function, and use graphs and tables to investigate its behavior.

     f. Recognize sequences as functions with domains that are whole numbers.

     g. Explore rates of change, comparing constant rates of change (i.e., slope) versus variable rates of change. Compare rates of change of linear, quadratic, square root, and other function families.

MM1G2. Students will understand and use the language of mathematical argument and justification.

     a. Use conjecture, inductive reasoning, deductive reasoning, counterexamples, and indirect proof as appropriate.

     b. Understand and use the relationships among a statement and its converse, inverse, and contrapositive.

FUNCTIONS

KEY IDEAS

1. Every function can be classified as a member of a "family." The "parent" of a function family is the most basic representation of the family. These are graphs of some basic **parent functions**.

Linear Function
$f(x) = x$
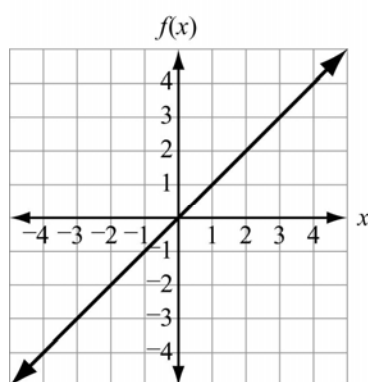
Absolute Value Function
$f(x) = |x|$
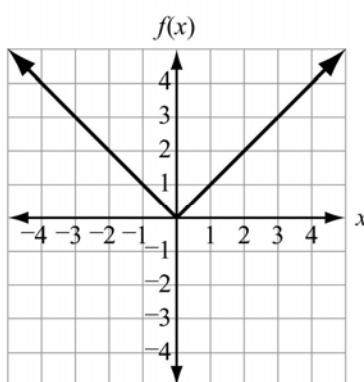
Quadratic Function
$f(x) = x^2$
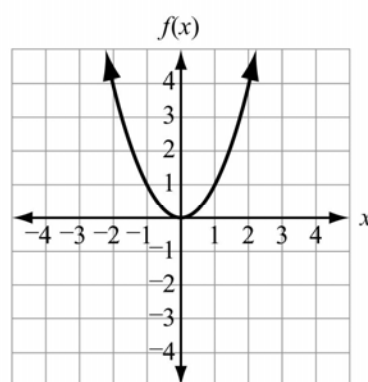
Cubic Function
$f(x) = x^3$
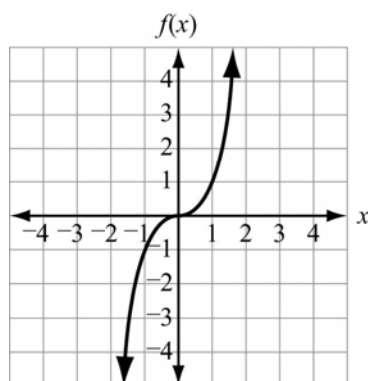
Reciprocal Function
$f(x) = \frac{1}{x}$
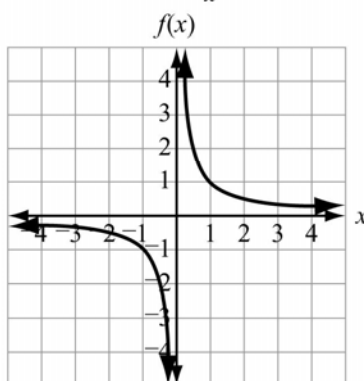
Square Root Function
$f(x) = \sqrt{x}$
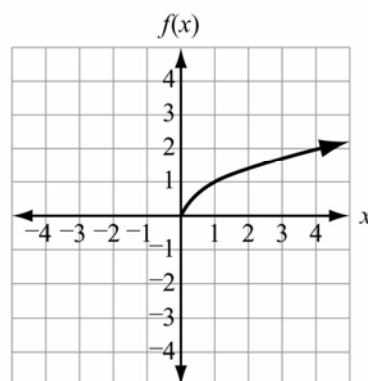
Parent functions can be transformed into many new functions. By studying the characteristics of a family of functions, you can determine how a parent graph is transformed.

Example:

Graph $f(x) = x + 2$, $f(x) = x^2 + 2$, and $f(x) = |x| + 2$. Describe how adding 2 to each parent function value affects the corresponding parent graph.

Solution:

$f(x) = x + 2$	$f(x) = x^2 + 2$	$f(x) = |x| + 2$
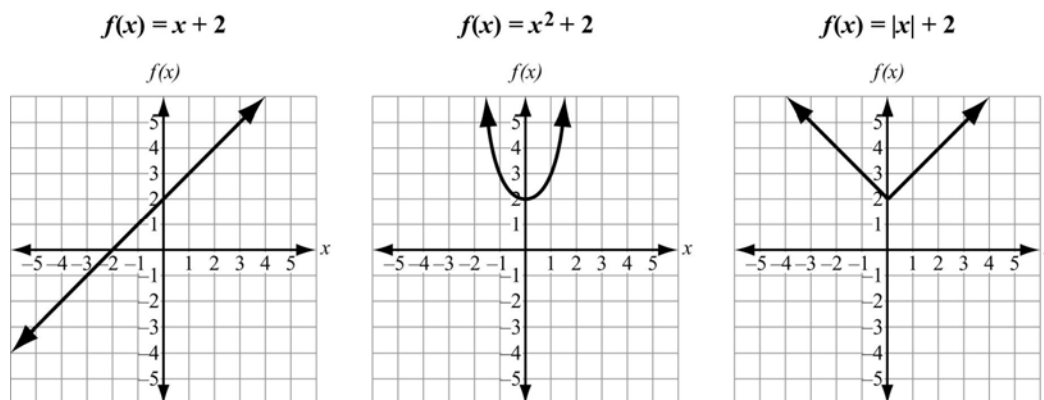
When 2 is added to each parent function, the corresponding parent graphs are vertically translated up 2 units.

Example:

Graph $f(x) = 5x$, $f(x) = 5x^2$, and $f(x) = 5|x|$. Describe how multiplying each parent function by 5 affects the corresponding parent graph.

Solution:

$f(x) = 5x$	$f(x) = 5x^2$	$f(x) = 5|x|$
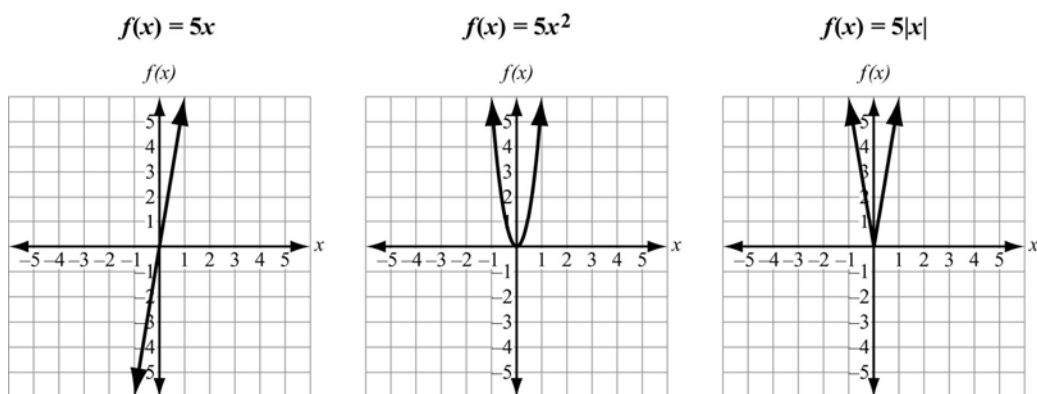
When 5 is multiplied to each parent function, the corresponding parent graphs become more narrow, otherwise known as vertically stretched.

Example:

Compare the graph of the parent function $f(x) = |x|$ with the graph of $f(x) = -|x|$. Then determine what the graph of the parent function $f(x) = x^2$ will look like, without graphing, when multiplied by -1: $f(x) = -(x^2)$.

Solution:

Multiplying the parent function $f(x) = |x|$ by -1 reflects the parent function across the x-axis.
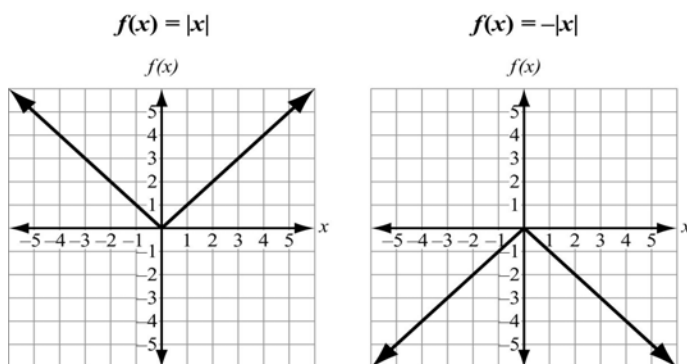
So, the graph of $f(x) = -(x^2)$ is the reflection of the parent function $f(x) = x^2$ reflected across the x-axis.

2. A function is an **even function** if $f(x) = f(-x)$ for all values in its domain. The graph of an even function is **symmetric** with respect to the y-axis. This means the graph is unchanged when reflected across the y-axis. The graph to the right shows an even function.
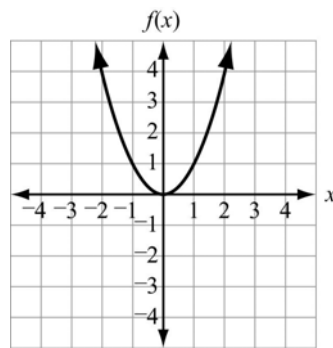
Even Function

A function is an **odd function** if $f(-x) = -f(x)$ for all values in its domain. The graph of an odd function is symmetric about the origin. This means that the graph is unchanged when reflected across both the x- and y-axes. The graph to the right shows an odd function.

Odd Function

16
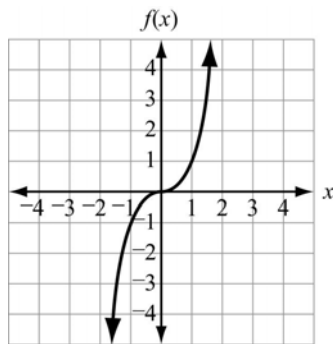
3. When given a table of values, you can either examine the ordered pairs or plot the ordered pairs to see if the graph is odd, even, or neither.

    Example:

    The table at the right shows the values of a function.
    Determine whether the function is odd, even, or neither.

x	$f(x)$
3	9
2	4
1	1
0	0
−1	1
−2	4
−3	9

    Solution:

    Examine the ordered pairs:
    $f(3) = 9; f(-3) = 9$
    $f(2) = 4; f(-2) = 4$
    $f(1) = 1; f(-1) = 1$

    Since $f(x) = f(-x),$ the function is an even function.

    Example:

    The table at the right shows the values of a function.
    Determine whether the function is odd, even, or neither.

x	$f(x)$
3	27
2	8
1	1
0	0
−1	−1
−2	−8
−3	−27

    Solution:

    Examine the ordered pairs:
    $f(-3) = -27; -f(3) = -27$
    $f(-2) = -8; -f(2) = -8$
    $f(-1) = -1; -f(1) = -1$

    Since $f(-x) = -f(x),$ the function is an odd function.

4. ***Function notation*** provides an efficient way to define and communicate functions. A function can be described using function notation: $f(x)$. This notation is read out loud as "f of x." It represents the output of the function f given the input value x.

    Example:

    The equation $y = x^3$ written in function notation is $f(x) = x^3$.

5. Functions have three parts: (i) a ***domain***, which is the set of inputs for the function, (ii) a ***range***, which is the set of outputs, and (iii) some ***rule or statement of correspondence*** indicating how each input determines a unique output.

6.   The domain and rule of correspondence determine the range of a function.

Example:

A company makes plastic cubes with sides that have lengths that are 1, 2, 3, or 4 inches long. The rule of correspondence for the function is $f(x) = x^3$, which represents the relationship between x, the length of the side of a cube, and $f(x)$, the volume of the cube. The domain would be $\{1, 2, 3, 4\}$ and the range would be $\{1, 8, 27, 64\}$.

7.   Graphs are geometric representations of functions.

Example:

This graph represents the function $f(x) = x^3$ from the example in Key Idea **#6**.
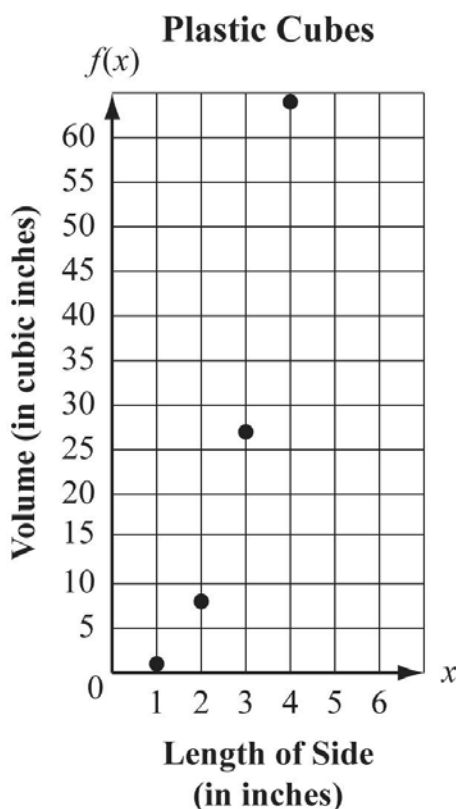
Note that the points start at 1, which is the smallest length of a side of a plastic cube. They are not connected because that would imply that the lengths of the cubes could include numbers in between the whole numbers that are given in the context of the problem. This is called a ***discrete*** function.

Example:

In one phase of a video game, a player is able to travel a distance, in miles, based on x, the number of seconds he is able to stay on a rocket. This graph represents the function $f(x) = x^3$, the relationship between the time, in minutes, on the rocket and the distance traveled.
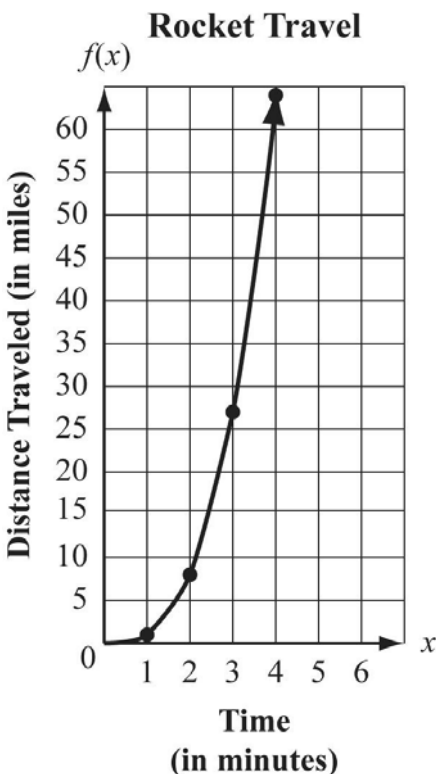
Note that the graph starts at 0 and the dots are connected. Time is continuous, so the graph should be a continuous (curved) line. This is called a ***continuous*** function.

8. Functions are equal if they have the same domain and rule of correspondence.

Example:

The functions $f(x) = x^3$ with the domain $\{1, 2, 3, 4, 5, 6\}$ and $g(x) = x^3$ with the domain $\{1, 2, 3, 4, 5, 6\}$ are equal.

9. The variables used to represent domain values, range values, and the function as a whole are arbitrary. Changing variable names does not change the function.

10. Functions that are nonlinear, such as quadratic, cubic, absolute-value, or square root functions, do not have a constant rate of change. Nonlinear functions have a ***variable rate of change***. This means that the change in the $f(x)$-value divided by the corresponding change in the x-value of a function over its entire domain is *not* always the same.

11. Consider the related functions whose relationships are specified by these equations and
     that each has the domain of the set of all real numbers.

           (i) $y = x^2$ **(ii)** $y = 5x^2$ **(iii)** $y = 6x^2$

     Each of these equations has the form $y = ax^2$, where a is a constant real number. In each of
     these situations, it is said that the y **varies directly** as the square of x.

12. The **vertex** of a quadratic equation is its highest or lowest point on a graph.

Important Tips for Working with Functions

- Begin exploration of a new function by generating a table of values using a variety of
  numbers from the domain. Decide, based on the context, what kinds of numbers can
  be in the domain, and make sure to choose negative numbers or numbers expressed as
  fractions or decimals if such numbers are included in the domain.

- Do extensive graphing by hand.

- Be extremely careful in the use of language. Always use the name of the function. For
  example, use f to refer to the function as a whole and use $f(x)$ to refer to the output
  when the input is x. For example, when language is used correctly, a graph of the
  function f in the x, y-plane is the graph of the equation $y = f(x)$, since only those
  points of the form (x, y) where the y-coordinates are equal to $f(x)$ are graphed.

REVIEW EXAMPLES

1) This graph represents the function $f(x) = -x^2 + 2x + 3$.
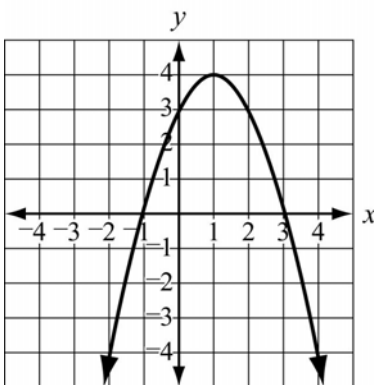
a. Identify the domain and the range of the function.
b. Identify the coordinates of the vertex. State whether the function has a maximum or
    minimum value.
c. Identify the zeros of the function and explain what they are.

Solution:

    a.   The domain is the set of all real numbers. The range is the set of numbers such that $y \leq 4$.

    b.   The coordinates of the vertex are (1, 4). The graph of the function is a parabola that opens downward, so it has a maximum value.

    c.   The zeros are the points where the graph crosses the x-axis. It crosses at $x = -1$ and $x = 3$. That means the value of the function is zero at $x = -1$ and $x = 3$.

2) A mail order company charges shipping based on the total weight of all the items purchased by a customer.

- The charge to ship items that weigh less than 3 pounds is $5.
- The charge to ship items that weigh at least 3 pounds but less than 6 pounds is $10.
- The charge to ship items that weigh at least 6 pounds but less than 9 pounds is $15.
- The charge to ship items that weigh at least 9 pounds but less than 12 pounds is $20.
- The charge to ship items that weigh at least 12 pounds but less than 15 pounds is $25.
- The pattern for charging continues.

This graph shows a function that represents the relationship between the total weight of all the items purchased by a customer and his or her shipping charges.

Mail Order Shipping Charges
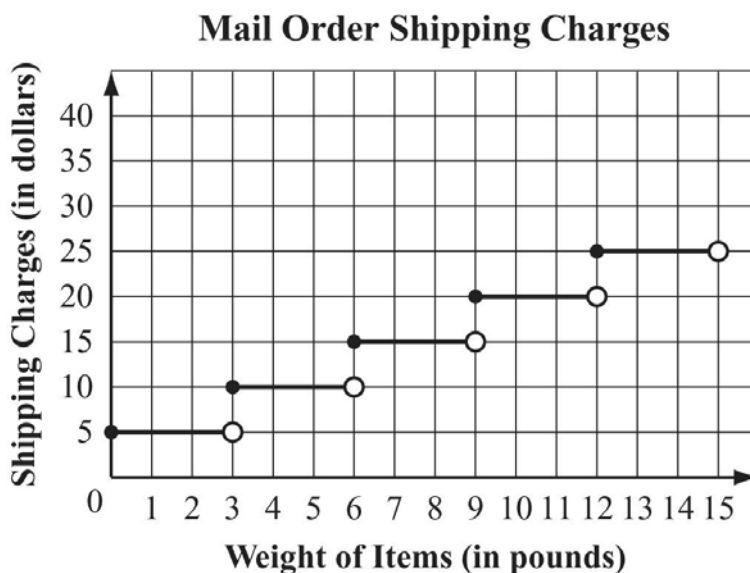
    a.   What is the domain of the function? Explain what the domain is in the context of the problem.

    b.   What is the range of the function? Explain what the range is in the context of the problem.

    c.   What is the charge to ship items weighing a total of $3\frac{1}{2}$ pounds?

Solution:

a. The domain is any positive real number. The domain is the total number of pounds of the items purchased by a customer.

b. The range is $\{\$5, \$10, \$15, \$20, \$25, \ldots\}$. The range is the amount of shipping paid by a customer. Note that the *y*-intercept of the function has a solid dot even though you would not ship something that weighs 0 pounds.

c. The charge is $10.

3) This table shows the total number of paper airplanes Gina made after school over time.

Gina's Paper Airplanes

Time (in minutes)	0	5	10	15	20	25	30
Total Number of Paper Airplanes	0	5	10	15	23	31	39

a. What is Gina's average rate of making paper airplanes during the first 15 minutes she made them?

b. What is Gina's average rate of making paper airplanes during the last 15 minutes she made them?

Solution:

a. $\dfrac{15-0}{15-0} = \dfrac{15}{15} = 1$ airplane per minute.

b. $\dfrac{39-15}{30-15} = \dfrac{24}{15} = 1.6$ airplanes per minute.

*Note***:** This is a nonlinear function. It has a *variable rate of change*.

EOCT Test Prep Items

1) Which statement best describes what is being modeled by this graph?

Wyatt's Jog
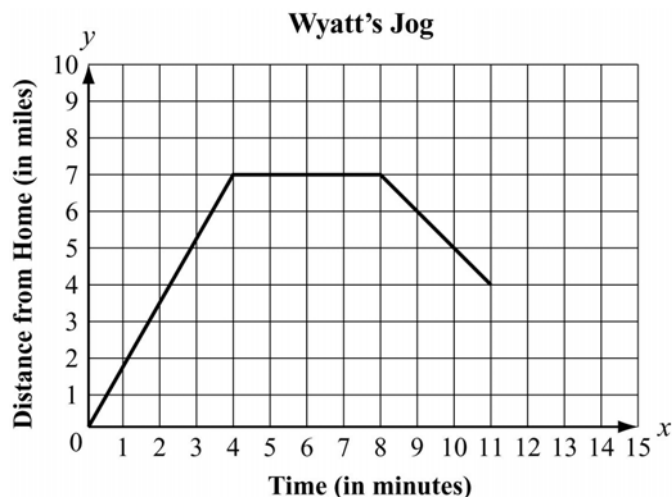
A. Wyatt started from a standstill, gradually picked up speed, jogged at a constant rate for 4 minutes, gradually slowed down, and stopped.
B. Wyatt began jogging at a constant rate and increased his pace steadily until coming to a complete stop after jogging for 11 minutes.
C. Wyatt jogged at a steady pace for 4 minutes, took a 4-minute break, walked at a steady pace for 3 minutes, and stopped.
D. Wyatt jogged uphill for 4 minutes, jogged on a flat surface for 4 minutes, jogged downhill for 3 minutes, and stopped.

[Key: C]

2) Heather is taking a turn playing a game.

- **If she answers the first question correctly, she is awarded 2 points.**
- **If she answers the second question correctly, she is awarded 4 points.**
- **If she answers the third question correctly, she is awarded 6 points.**

Heather's turn and this pattern will continue until she is not able to answer a question correctly. Heather answers *n* questions correctly during her turn. Which function can be used to calculate the total number of points that she was awarded?

A. $f(n) = n^2 + 2n$
B. $f(n) = n^2 + n$
C. $f(n) = n^2 + 2$
D. $f(n) = n^2 + 1$

[Key: B]

MATHEMATICAL ARGUMENT AND JUSTIFICATION

KEY IDEAS

1. *Inductive reasoning* is the process of looking for a pattern and using that pattern to make a generalization.

2. *Deductive reasoning* is the process of coming to a logical conclusion based on an accepted hypothesis.

3. A *counterexample* is an example that shows a conjecture to be false. You need only one counterexample to prove that a conjecture is not true.

4. An *indirect proof* is based on eliminating all possible conclusions except for one, resulting in the understanding that the one that is left must be true. It is a proof based on contradiction.

5. A *statement* is a sentence that is either true or false, but not both.

6. By definition, a statement is true or false; whether a statement is true or false is called the *truth value* of the statement.

7. A *compound statement*, or *compound proposition*, is a new statement formed by putting two or more statements together.

8. If p and q are statements, then the statement "if p, then q" is the *conditional statement*, or implication, with *hypothesis* p and *conclusion* q. The variables p and q are called statement or *propositional variables*.

9. If two propositional forms result in statements with the same truth value for all possible cases of substituting statements for the propositional variables, the forms are *logically equivalent*.

10. Two propositional forms are *not logically equivalent* if there exists some group of statements that can be substituted into the propositional forms so that the two statements corresponding to the two forms have different truth values.

11. If p and q are statements, then the statement "p if and only if q" is called a *biconditional* statement and is logically equivalent to the statement "if q, then p" and "if p, then q."

12. If a statement is "p," then the *negation* of the statement is "not p."

13. In general, a ***conditional statement*** is a statement that can be expressed in "if . . . then" form.

    - The ***converse*** of a conditional statement is the new statement obtained by exchanging the hypothesis and the conclusion.

    - The ***inverse*** of a conditional statement is the new statement obtained by negating the hypothesis and the conclusion.

    - The ***contrapositive*** of a conditional statement is the new statement obtained by both negating and exchanging the hypothesis and the conclusion.

    Example:

Statement	If two angles are complementary, then the sum of the measures of the two angles is 90°.
Converse	If the sum of the measures of two angles is 90°, then the two angles are complementary.
Inverse	If two angles are not complementary, then the sum of the measures of the two angles is not 90°.
Contrapositive	If the sum of the measures of two angles is not 90°, then the two angles are not complementary.

14. To say that a conditional statement is true means that whenever the hypothesis is true, then the conclusion is also true; to say that a conditional statement is false means that the hypothesis is, or can be, true while the conclusion is false.

REVIEW EXAMPLES

1) The vertex of $\angle B$ in $\triangle ABC$ forms a vertical angle with $\angle GBH$, and that $m\angle GBH = 64°$ and $m\angle BAC = 73°$.

    What conclusions can you draw and support about $\triangle ABC$ from this information?

Solution:

    The accepted facts are:
    - $\angle B$ in $\triangle ABC$ forms a vertical angle with $\angle GBH$.
    - $m\angle GBH = 64°$
    - $m\angle BAC = 73°$

    Use deductive reasoning to draw conclusions about $\triangle ABC$.
    - Since $\angle B$ in $\triangle ABC$ forms a vertical angle with $\angle GBH$, then $m\angle ABC = 64°$, because vertical angles are congruent.

- Since $m\angle BAC = 73°$ and $m\angle ABC = 64°$, then $m\angle ACB = 43°$, because the sum of the angle measures of any triangle is 180º and $m\angle ACB = 180° - 73° - 64°$.
- Since the angle measures of $\triangle ABC$ are all acute angles, 73º, 64º, and 43º, then $\triangle ABC$ is an acute triangle.

2) Samuel wrote this statement.

"The number of feet of the perimeter of a rectangle is always greater than the number of square feet in its area."

What is a counterexample to Samuel's statement?

Solution:

If a rectangle has dimensions of 5 feet by 4 feet, it has a perimeter of 18 feet and an area of 20 square feet. This is a counterexample because it gives an example where Samuel's statement is false. It only takes one counterexample to show that a statement is false.

EOCT Test Prep Items

1) **If David goes to the mall, then his brother will go to the movies. David's brother did not go to the movies.**

   Assuming that these two statements are true, what conclusion can be drawn?

   A. David went to the mall.
   B. David went to the movies.
   C. David did not go to the mall.
   D. David did not go to the movies.

[Key: C]

2) **Ella factored the first five out of ten trinomials on a test, and each one factored into a pair of binomials. She made this statement.**

   "All of the trinomials on this test will factor into a pair of binomials."

   Which word or phrase best describes Ella's statement?

   A. counterexample
   B. inductive reasoning
   C. deductive reasoning
   D. conditional statement

[Key: B]

SEQUENCES

KEY IDEAS

1. A *sequence* is an ordered list of numbers, pictures, letters, geometric figures, or just about any object you like. Each number, figure, or object is called a *term* in the sequence. For convenience, the terms of sequences are often separated by commas.

2. Each term in a sequence is typically represented by a_n, where n is a whole number that represents the location of that term in the sequence.

   Example:

   Consider the sequence 6, 3, 0, –3, –6, –9, …. The first term in the sequence is 6, and it is represented by a_1. The second term in the sequence is 3, and it is represented by a_2. The nth term of the sequence is represented by a_n.

3. *Finite* sequences contain a finite (bounded) number of terms.

4. *Infinite* sequences contain an infinite (unbounded) number of terms. The sequence continues in the same pattern to infinity.

5. The three dots within a list of terms in some sequences is called an *ellipsis* and indicates that some of the terms are missing. An ellipsis is necessary at the end of an infinite sequence to indicate that the sequence goes on and on to infinity.

6. Some sequences follow predictable patterns, though the pattern might not be immediately apparent. Other sequences have no pattern at all.

   Example:

   2, 4, 6, 8, 10, . . .
   This sequence has a pattern. It is even numbers in increasing order.

   Example:

   7, 9, 3, 5, 4, 8
   This sequence does not appear to have a pattern.

7. In looking for patterns in sequences, it is useful to look for a pattern in how each term relates to the previous one. If there is a consistent pattern in how each term relates to the previous one, it is convenient to express this pattern using a ***recursive definition*** for the sequence. A recursive definition gives the first term and a formula for how the nth term relates to the $(n-1)^{th}$ term.

   Example:

   Sequence: 3, 6, 9, 12, 15, . . .
   Recursive definition: $t_1 = 3, \; t_n = t_{n-1} + 3$

8. Another way to define a sequence uses a ***closed form definition*** that indicates how to determine the nth term directly.

   Example:

   Sequence: 3, 6, 9, 12, 15, . . .
   Closed form definition: $t_n = 3n, \;$ for $n = 1, 2, 3, \ldots$

REVIEW EXAMPLES

1) Consider this sequence.

   $5, 7, 11, 19, 35, 67, \ldots$

   a. Is this a finite sequence or an infinite sequence?
   b. What is a_1? What is a_3?
   c. What is the domain of the sequence? What is the range?

Solution:

   a. The ellipsis at the end of the sequence indicates that it is an infinite sequence.
   b. a_1 is 5, a_3 is 11.
   c. The domain is $\{1, 2, 3, 4, 5, 6 \ldots\}$, and the range is $\{5, 7, 11, 19, 35, 67, \ldots\}$.

   Note that this sequence has a pattern that can be expressed using the recursive definition $a_1 = 5, \; a_n = 2a_{n-1} - 3$.

2) The function $f(n) = -(1-4n)$ represents a sequence. Create a table showing the first five terms in the sequence. Identify the domain and range of the function.

Solution:

n	1	2	3	4	5
$f(n)$	3	7	11	15	19

Since the function is a sequence, the domain would be n, the number of each term in the sequence. The set of numbers in the domain can be written $\{1, 2, 3, 4, 5, \ldots\}$. Notice that the domain is an infinite set of numbers, even though the table lists only the first 5.

The range is $f(n)$ or (a_n), the output numbers that result from applying the rule $-(1-4n)$. The set of numbers in the range, which is the sequence itself, can be written $\{3, 7, 11, 15, 19, \ldots\}$. This is also an infinite set of numbers, even though the table lists only the first 5.

EOCT Test Prep Items

1) **These are the first four steps of a dot pattern.**

Step 1   Step 2   Step 3   Step 4
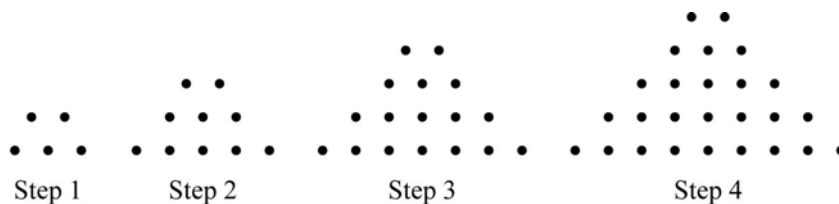
The pattern continues. Which function represents the number of dots in Step n?

A. $f(n) = n^2 + n - 5$
B. $f(n) = n^2 + n + 3$
C. $f(n) = n^2 + 5n - 1$
D. $f(n) = n^2 + 2n + 2$

[Key: D]

2) The first term in this sequence is –3.

n	1	2	3	4	5	. . .
a_n	–3	4	23	60	121	. . .

Which function represents the sequence?

A. $f(n) = n^2 - 4$

B. $f(n) = n^3 - 4$

C. $f(n) = -n^3 + 4$

D. $f(n) = -n^2 + 4$

[Key: B]

Unit 2: Algebra Investigations

The focus of this unit is the development of students' abilities to read and write the symbolically intensive language of algebra. Students develop skills in adding, subtracting, multiplying, and dividing elementary polynomial, rational, and radical expressions. By using algebraic expressions to represent quantities in context, students understand algebraic rules as general statements about operations on real numbers. Work with products and the zero factor property are introduced.

KEY STANDARDS

MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.

    a. Simplify algebraic and numeric expressions involving square roots.
    b. Perform operations with square roots.
    c. Add, subtract, multiply, and divide polynomials.
    d. Expand binomials using the Binomial Theorem.
    e. Add, subtract, multiply, and divide rational expressions.
    f. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + y)(x - y) = x^2 - y^2$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

    f. Use area and volume models for polynomial arithmetic.

MM1A3. Students will solve simple equations.

    a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.

ALGEBRAIC EXPRESSIONS

KEY IDEAS

1.   An *algebraic expression* is an expression that contains at least one variable.

2.   A *polynomial* is an algebraic expression that contains one or more monomials separated by + or − signs. Each monomial is a **term** of the polynomial.

3.   To add or subtract polynomials, combine like terms. Recall that *like terms* have the same variable(s) with the same exponent(s).

   Example:

   $(7x^4 - 3x^2) + (9x^4 - 2x^2 + 6)$
   $= (7x^4 + 9x^4) + (-3x^2 - 2x^2) + 6$ One way is to use the Commutative and Associative Properties
                                              to reorder and regroup the terms.
   $= 16x^4 - 5x^2 + 6$ Combine like terms.

4.   Use the *distributive property* to multiply polynomials.

   Example:

   $4x(-3x^2 + 4x - 2) = 4x(-3x^2) + (4x)(4x) - (4x)(2)$ Distribute $4x$ to each term.
   $\qquad\qquad\qquad\quad = -12x^3 + 16x^2 - 8x$ Multiply coefficients and add exponents.

   Example:

   $(x - 4)(x + 7)$
   $= x^2 + 7x - 4x - 28$ Distribute x in the first parentheses to each term
                              in the second parentheses. Then distribute $- 4$
                              to each term in the second parentheses.
   $= x^2 + 3x - 28$ Combine like terms.

5.   These are the formulas for six special products that always result in a pattern.
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x-y)^2 = x^2 - 2xy + y^2$$
$$(x+y)(x-y) = x^2 - y^2$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

6.   To **factor** a polynomial means to write the polynomial as the product of factors. In the list of special products, the polynomial is the expression to the right of the equal sign and the factored form is the expression to the left of the equal sign.

7.   To factor an expression, one or more of the following factoring methods may have to be performed. A variety of factoring methods can be used to factor an expression.

    - Factoring out a greatest common factor.
      $$2x^2 + 6x + 10 = 2(x^2 + 3x + 5)$$

    - Factoring by grouping.
      $$8 + xy + 4y + 2x = (2x+8) + (xy+4y) = 2(x+4) + y(x+4) = (2+y)(x+4)$$

    - Factoring a trinomial into two binomials using trial and error.
      $$x^2 + 2x - 15 = (x + \boxed{?})(x + \boxed{?}) = (x + (-3))(x + 5)$$

    - Factoring using patterns of special products.
      Difference of two squares: $x^2 - y^2 = (x+y)(x-y)$
      Square of a binomial: $(x+y)^2 = x^2 + 2xy + y^2$ **or** $(x-y)^2 = x^2 - 2xy + y^2$
      Cube of a binomial: $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ **or** $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

8.   The **Zero Product Property** states that if a and b are real numbers such that the product ab equals 0, then $a = 0$ or $b = 0$.

9.   The **Binomial Theorem** can be used to determine powers of binomials. This theorem is helpful in learning how to expand binomials, such as $(x+y)^n$, especially for large values of n. Without this theorem, $(x+y)$ would have to be multiplied by itself n times. For the EOCT, you will need to know how to expand binomials with n, at most, equaling 3.

The Binomial Theorem

Let n be a positive integer.

$$(x+y)^n = x^n + \frac{n}{1}x^{n-1}y^1 + \frac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \ldots y^n$$

- There are $n + 1$ terms.
- The first term is x^n and the last term is y^n.
- The sum of the exponents in each term is n.
- The exponent of x in any term is one less than the exponent of x in the preceding term.
- The exponent of y in any term is one more than the exponent of y in the preceding term.
- The coefficient of any term in an expansion is $\dfrac{n!}{(\text{exponent of }x)!(\text{exponent of }y)!}$.

The Binomial Theorem can also be written in **factorial notation**.

$$(x+y)^n = x^n + \frac{n!}{(n-1)!1!}x^{n-1}y + \frac{n!}{(n-2)!2!}x^{n-2}y^2 + \ldots y^n$$

Example:

Expand $(5x - y)^3$.

Solution:

In the binomial $(5x - y)^3$, $n = 3$. So, the expansion will have $n + 1$, or 4, terms.

$$(5x-y)^3 = \left[5x+(-y)\right]^3 \qquad \text{Rewrite expression in the form } (x+y)^n.$$
$$\text{Follow the Binomial Theorem.}$$

$$= (5x)^3 + \frac{3\cdot 2\cdot 1}{2\cdot 1\cdot 1}(5x)^2(-y)^1 + \frac{3\cdot 2\cdot 1}{1\cdot 2\cdot 1}(5x)^1(-y)^2 + (-y)^3$$
$$= 125x^3 - 75x^2y + 15xy^2 - y^3$$

Notice that there are 4 terms, the variable for the first term is x^3, the variable for the last term is y^3, and the sum of the exponents in each term is 3.

10. *Equivalent expressions* give the same numerical value output for every number in the domain for the expressions.

Example:

Consider the sequence: 3, 6, 9, 12, 15. . .. There are different ways to look at the pattern of the sequence.

One way is to say each number in the sequence is 3 times the number of its step. The fourth step would be 3 times 4 or 12. This can be written as the algebraic expression $3n$, where n stands for the number of the step.

Another way is to say each number in the sequence is 3 added to 3 times the number of the step before it. The fourth step would be 3 added to 3 times 3 (the number of the step before it) or 12. This can be written as the algebraic expression $3 + 3(n-1)$, where n stands for the number of the step.

The expressions $3n$ and $3 + 3(n-1)$ are equivalent expressions because they represent the same sequence. They give the same numerical value output for every number in the domain for the expressions.

11.   No conclusion can be drawn when substituting only one numerical value into two different algebraic expressions and getting the same result for both expressions.

12.   An important conclusion can be drawn when substituting one numerical value into two different algebraic expressions and getting *different* results for the two expressions. The expressions are **not** equivalent. It takes only one counterexample to conclude that they are not equivalent.

13.   For algebraic expressions that define *linear functions*, it is sufficient to check that two numerical values substituted into the expressions give the same result. This is because the graph of the function is a line and two points are enough to determine a line.

REVIEW EXAMPLES

1) Use a geometric figure to find the product of $(x+5)(x+2)$.

Solution:

Model the product using a rectangle. First draw a rectangle. Label the length $(x + 5)$. Label the width $(x + 2)$.

$x + 5$

$x + 2$ ▭
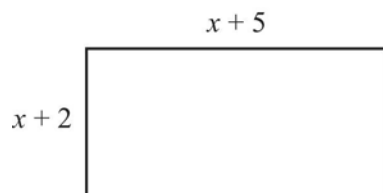
The area of this rectangle is the product of the length and the width: $(x+5)(x+2)$. Now divide the rectangle into four regions, as shown below.

	x	$+$	5
x			
$+$			
2			
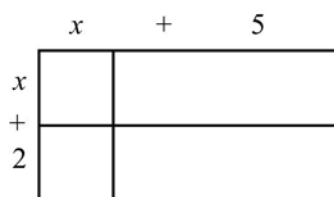
Find the area of each region.

	x	$+$	5
x	x^2		$5x$
$+$			
2	$2x$		10
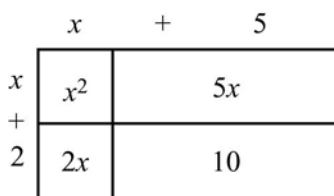
Then find the sum of the areas to find the total area of the rectangle.

$$A = 10 + 2x + 5x + x^2$$
$$= x^2 + 7x + 10$$

So, $(x+5)(x+2) = x^2 + 7x + 10$.

2) This rectangle shows the floor plan of an office.

The shaded part of the plan is an area that is getting new tile. Write an algebraic expression that represents the area of the office that is getting new tile.

Solution:

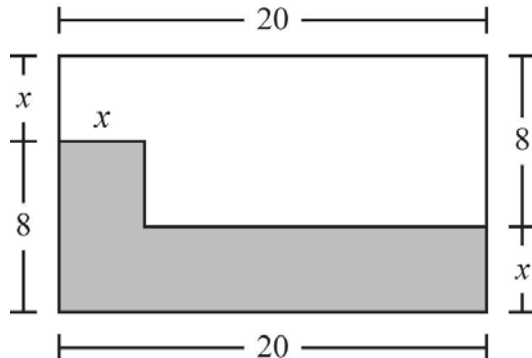It is helpful to divide the figure into rectangles with dimensions that will make it easy to calculate the area. There is more than one way to do this, but we will use this division of the rectangle.
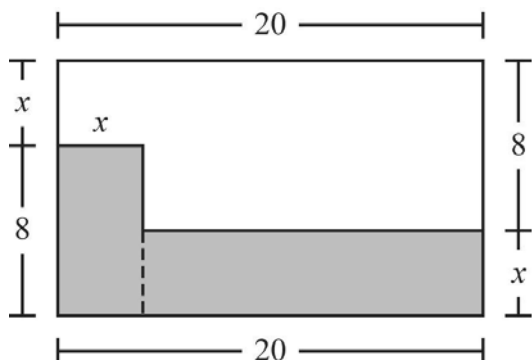
Now we have one shaded rectangle that is 8 units long and x units wide and another shaded rectangle that is $20 - x$ units long and x units wide.

$$(8)(x) + (20 - x)(x) \quad \text{Add the products of the lengths and widths}$$
$$\text{of the rectangles.}$$
$$= 8x + 20x - x^2 \quad \text{Multiply.}$$
$$= 28x - x^2 \quad \text{Combine like terms.}$$

The area, in square units, is represented by $28x - x^2$.

EOCT Test Prep Items

1) **A train travels at a rate of $(4x+5)$ miles per hour. How many miles can it travel at that rate in $(x-1)$ hours?**

   A. $3x-4$ miles
   B. $5x-4$ miles
   C. $4x^2+x-5$ miles
   D. $4x^2-9x-5$ miles

   [Key: C]

2) **Taylor and Susan each have a box that is in the shape of a cube. The edges of Taylor's box are each x cm in length. The edges of Susan's box are 4 cm longer than on Taylor's cube. What binomial expansion represents the volume of Susan's box?**
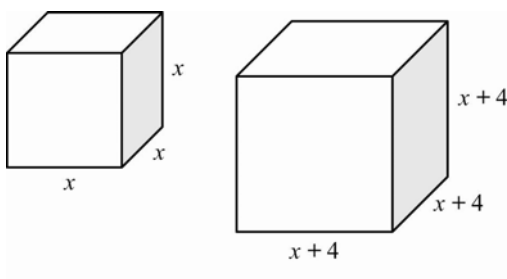
   A. $(x^2+8x+16)\,\text{cm}^3$
   B. $(x^3+12x^2+48x+64)\,\text{cm}^3$
   C. $(x^3+768x^2+192x+4)\,\text{cm}^3$
   D. $(x^4+12x^3+96x^2+256x+256)\,\text{cm}^3$

   [Key: B]

3) **What is the product of the expression represented by the model below?**
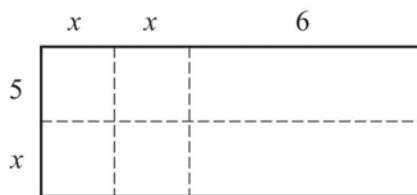
   A. $3x+11$
   B. x^3+30
   C. $2x^2+10x+36$
   D. $2x^2+16x+30$

   [Key: D]

RATIONAL EXPRESSIONS

KEY IDEAS

1. A **rational expression** is the ratio of two polynomials, $\dfrac{P_1}{P_2}$, where $P_2 \neq 0$.

2. When the numerator and denominator in the rational expression have no common factors other than 1, then the expression is in ***simplest form***.

3. To multiply rational expressions:
   - factor each numerator and denominator
   - divide out common factors
   - multiply the numerators and multiply the denominators
   - simplify the product if it is not in simplest form

   Example:

   $$\frac{4x}{8x+2} \cdot \frac{2x+2}{4x+4}$$

   $$= \frac{(2)(2)(x)}{(2)(4x+1)} \cdot \frac{(2)(x+1)}{(2)(2)(x+1)} \qquad \text{Factor.}$$

   $$= \frac{x}{4x+1} \cdot \frac{1}{1} \qquad\qquad\qquad \text{Divide out common factors.}$$

   $$= \frac{x}{4x+1} \qquad\qquad\qquad\qquad \text{Multiply the numerators. Multiply the denominators.}$$

4. To divide rational expressions:
   - multiply the first expression by the reciprocal of the second expression
   - follow the rules for multiplying rational expressions

   Example:

   $$\frac{5x}{15x^2} \div \frac{25x}{x^2}$$

   $$= \frac{5x}{15x^2} \cdot \frac{x^2}{25x} \qquad\qquad\qquad \text{Multiply by the reciprocal.}$$

   $$= \frac{(5)(x)}{(3)(5)(x)(x)} \cdot \frac{(x)(x)}{(5)(5)(x)} \qquad \text{Factor.}$$

   $$= \frac{1}{3} \cdot \frac{1}{25} = \frac{1}{75} \qquad\qquad\quad \text{Divide out common factors and multiply.}$$

5.   To add or subtract rational expressions with like denominators, combine like terms in the numerator and keep the denominator the same.

   Example:

   $$\frac{4x}{9x^2} - \frac{2x-3}{9x^2} = \frac{4x}{9x^2} - \frac{(2x-3)}{9x^2}$$ Rewrite using parentheses around the numerator of the second fraction.

   $$= \frac{4x - 1(2x-3)}{9x^2}$$ Rewrite with -1 multiplied by the parentheses.

   $$= \frac{4x - 2x + 3}{9x^2}$$ Distribute -1.

   $$= \frac{2x+3}{9x^2}$$ Combine like terms. Keep the denominator the same.

6.   To add or subtract rational expressions with unlike denominators:
   - find the least common denominator (LCD) of each expression by writing the prime factorization of each denominator
   - use the LCD to write equivalent expressions
   - follow the rules for adding and subtracting rational expressions with like denominators

   Example:

   $$\frac{6}{3y} + \frac{1}{3y+6} = \frac{6}{(3)(y)} + \frac{1}{(3)(y+2)}$$ Factor. The LCD is $(3)(y)(y+2)$.

   $$= \frac{6}{(3)(y)}\left(\frac{y+2}{y+2}\right) + \frac{1}{(3)(y+2)}\left(\frac{y}{y}\right)$$ Build each fraction by multiplying by a fraction

   equal to 1 that will make the denominators common.

   $$= \frac{6y+12}{3y(y+2)} + \frac{y}{3y(y+2)}$$

   $$= \frac{7y+12}{3y(y+2)}$$ Combine like terms in the numerators and

   keep the denominator the same.

   Be sure answer is in simplest form.

REVIEW EXAMPLES

1) Greg hiked 10 miles from a ranger station to a campground on Monday. On Tuesday he hiked back to the ranger station. The campground was uphill from the ranger station, so his average rate of speed to the campground was 2 miles per hour slower than it was to the ranger station.

   Let r represent Greg's average rate of speed to the ranger station. Write an expression that represents the total time, in hours, that Greg hiked.

Solution:

   First write an expression that represents the time it took Greg to hike to the campground and an expression that represents the time it took him to hike to the ranger station. Add the two together and simplify. Remember that distance = rate • time, so time $= \dfrac{\text{distance}}{\text{rate}}$.

   The time to the campground was $\dfrac{10}{r-2}$, and the time to the ranger station was $\dfrac{10}{r}$.

$$\frac{10}{r-2}+\frac{10}{r}=$$ The LCD is $r(r-2)$.

$$\frac{10}{(r-2)}\left(\frac{r}{r}\right)+\frac{10}{r}\left(\frac{r-2}{r-2}\right)=$$ Build each fraction by multiplying by

   a fraction equal to 1 that will make the denominators common.

$$\frac{10r}{r(r-2)}+\frac{10r-20}{r(r-2)}=$$ Simplify the numerators.

$$\frac{20r-20}{r(r-2)}$$ Combine like terms in the numerators

   and keep the same denominator.

   Be sure answer is in simplest terms.

   The total time Greg hiked can be represented by $\dfrac{20r-20}{r(r-2)}$.

2) Shannon had x cookies. David had 5 more cookies than Shannon.

- Shannon ate half of her cookies.
- David ate one-third of his cookies.

Write an algebraic expression in simplest form that could represent the total number of cookies that David and Shannon ate.

Solution:

Since x represents the number of cookies Shannon had, then $x + 5$ can represent the number of cookies David had.

Now represent the number of cookies that each one ate by multiplying each expression by the fractional part of the cookies eaten.

The number of cookies that Shannon ate was $\frac{1}{2}x$ or $\frac{x}{2}$.

The number of cookies that David ate was $\frac{1}{3}(x+5)$ or $\frac{x+5}{3}$.

To get the total number of cookies eaten, add the two fractional parts together.

$$\frac{x}{2} + \frac{x+5}{3}$$ Find the common denominator.

$$= \frac{x}{2}\left(\frac{3}{3}\right) + \frac{x+5}{3}\left(\frac{2}{2}\right)$$ Build the fractions by multiplying each fraction

by a fraction that equals 1 that will make the denominator common.

$$= \frac{3x}{6} + \frac{2x+10}{6}$$ Multiply.

$$= \frac{5x+10}{6}$$ Combine numerators and keep denominator the same.

Be sure the answer is in simplest form. This is in simplest form because the numerator and the denominator do not have any factors in common.

EOCT Test Prep Items

1) **Which expression represents the area of a rectangle given that the length is $\dfrac{4d-12}{d^2-9}$ and the width is $3d+9$?**

   A. 7

   B. 12

   C. $\dfrac{4}{3(d+3)^2}$

   D. $\dfrac{7d-3}{d^2-9}$

   [Key: B]

2) **Which expression is equivalent to $3 \div \dfrac{9}{27-x}$?**

   A. $\dfrac{27-x}{3}$

   B. $\dfrac{9-x}{9}$

   C. $\dfrac{3}{9-x}$

   D. $\dfrac{3}{27-x}$

   [Key: A]

RADICAL EXPRESSIONS

KEY IDEAS

1. The opposite process of squaring a number is finding its **square root**. The square root of any number, x, is the number that when multiplied by itself equals x. Recall that the symbol $\sqrt{\ }$ is called a **radical symbol**, and the number under the radical is called a **radicand**.

2. **Product Property of Radicals:** for all real numbers x and y, where $x > 0$ and $y > 0$, $\sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$.

    Example:

    $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

    Example:

    $\sqrt{x^4 y^3} = \sqrt{x^4} \cdot \sqrt{y^2} \cdot \sqrt{y}$ Rewrite as the product of the square roots of the factors.

    $= x^2 y \sqrt{y}$ Simplify.

3. **Quotient Property of Radicals:** for all real numbers x and y, where $x > 0$ and $y > 0$,

    $\sqrt{\dfrac{x}{y}} = \dfrac{\sqrt{x}}{\sqrt{y}}$.

    Example:

    $\sqrt{\dfrac{100}{16}} = \dfrac{\sqrt{100}}{\sqrt{16}} = \dfrac{10}{4} = \dfrac{5}{2}$

Example:

$$\sqrt{\frac{x^5}{y^6}} = \frac{\sqrt{x^5}}{\sqrt{y^6}}$$ Rewrite as a quotient of the square roots.

$$= \frac{\sqrt{x^4} \cdot \sqrt{x}}{\sqrt{y^6}}$$ Rewrite the numerator as the product of the square roots of the factors.

$$= \frac{x^2 \sqrt{x}}{y^3}$$ Simplify.

4.   To add or subtract radicals, the expressions must be like radicals, that is they must have the exact same radicand. Add or subtract the coefficients and keep the radicand the same.

Example:

$$2\sqrt{ab} + 5\sqrt{ab} = 7\sqrt{ab}$$

Example:

$$3\sqrt{4x^3} + 7x\sqrt{x}$$ At first it does not look like these can be combined.

$$= 3\sqrt{4} \cdot \sqrt{x^2} \cdot \sqrt{x} + 7x\sqrt{x}$$ Factor where possible.

$$= 6x\sqrt{x} + 7x\sqrt{x}$$ Simplify. Now there are like terms.

$$= 13x\sqrt{x}$$ Combine the like terms.

5.   The answer to a problem that involves radicals may be in radical form. If the problem involves finding a length or distance, a calculator may be needed to find the approximate value of the expression.

Example:

Cal was making a rectangular flag that was 10 inches by 16 inches and a rectangular flag that was 12 inches by 18 inches. He wanted to buy enough ribbon to attach a strip down the diagonal of each flag. What is the minimum whole number of inches of ribbon that Cal should buy?

Solution:

First find the diagonal of each flag by using the Pythagorean theorem.

First Flag	**Second Flag**
$a^2 + b^2 = c^2$	$a^2 + b^2 = c^2$
$10^2 + 16^2 = c^2$	$12^2 + 18^2 = c^2$
$100 + 256 = c^2$	$144 + 324 = c^2$
$c^2 = 356$	$c^2 = 468$
$c = \sqrt{356}$	$c = \sqrt{468}$

The sum of the lengths of the two diagonals is $\sqrt{356}$ inches $+ \sqrt{468}$ inches. Since we want to find out how many inches of ribbon Cal should buy, we need to use a calculator to get an approximate value for each square root.

$$\sqrt{356} \approx 18.9 \text{ inches}$$
$$\sqrt{468} \approx 21.6 \text{ inches}$$

Add and round up to the nearest whole inch of ribbon: $18.9 + 21.6 = 40.5$. Cal should buy 41 inches of ribbon.

REVIEW EXAMPLES

1) A square patio has an area of $98x^2$ square feet. Write an expression that represents the length of one side of the patio.

Solution:

To find the length of a side of a square where the area is given, take the square root of the area.

$$\sqrt{98x^2} = \sqrt{49} \cdot \sqrt{2} \cdot \sqrt{x^2} \quad \text{Rewrite as the product of the square roots of the factors.}$$
$$= 7x\sqrt{2} \quad\quad\quad\quad\quad\quad \text{Simplify.}$$

2) Katie set up a face painting booth at the school carnival. She started with a space in the shape of a square with a side length of 10 feet. She increased the length of the booth by x feet. The white area in this diagram shows the original shape of the booth, and the shaded area shows the extra part Katie added to it.
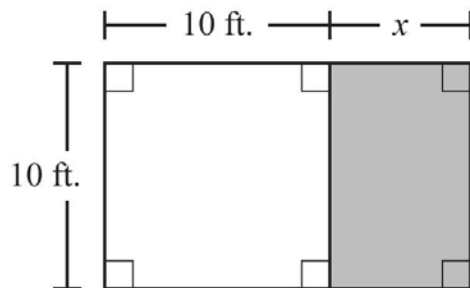
Katie plans to hang streamers diagonally across her booth. Write an expression that represents the length of the diagonal of the booth.

Solution:

To find the diagonal, in feet, of the booth, use the Pythagorean theorem using $10 + x$ for the length of the booth and 10 for the width.

$a^2 + b^2 = c^2$

$(x+10)^2 + 10^2 = c^2$ Rewrite the Pythagorean theorem using the representations of the length and the width of the booth.

$x^2 + 20x + 100 + 100 = c^2$ Square the two dimensions.

$c^2 = x^2 + 20x + 200$ Switch the order of the equation and simplify.

$c = \sqrt{x^2 + 20x + 200}$ Solve by taking the square root of each side.

Note that the expression under the radical will not factor and that there is no way to take the square root of this expression. This is the answer in simplest form.

EOCT Test Prep Items

1) **What value of x makes the equation $\sqrt{x} + 3 = 7$ true?**

   A. 4
   B. 10
   C. 16
   D. 21

[Key: C]

2) **A right isosceles triangle has a hypotenuse with a length represented by $4y$. Which expression represents the length of one of the legs of the triangle?**

   A. $2y$

   B. $4y$

   C. $y\sqrt{2}$

   D. $2y\sqrt{2}$

[Key: D]

Unit 3: Geometry Gallery

In this unit, students will explore, understand, and use the formal language of reasoning and justification. They will also use logical reasoning and proofs; prove conjectures through multiple forms of justification; explore angles, triangle inequalities, congruences, and points of concurrency; and apply properties to determine special quadrilaterals.

KEY STANDARDS

MM1G3. Students will discover, prove, and apply properties of triangles, quadrilaterals, and other polygons.
    a. Determine the sum of interior and exterior angles in a polygon.
    b. Understand and use the triangle inequality, the side-angle inequality, and the exterior-angle inequality.
    c. Understand and use congruence postulates and theorems for triangles (SSS, SAS, ASA, AAS, and HL).
    d. Understand, use, and prove properties of and relationships among special quadrilaterals: parallelogram, rectangle, rhombus, square, trapezoid, and kite.
    e. Find and use points of concurrency in triangles: incenter, orthocenter, circumcenter, and centroid.

ANGLE PROPERTIES AND RELATIONSHIPS OF ANGLES AND SIDES OF POLYGONS

KEY IDEAS

1. In a ***regular polygon***, all side lengths are congruent, and all angles are congruent.

2. The following information applies to ***interior and exterior angles*** of polygons:

   - The ***Interior Sum Theorem*** for triangles states that the sum of the measures of the three interior angles of a triangle always equals 180°.

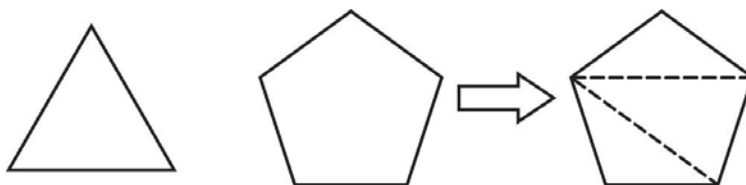   - The ***sum of the measures of the interior angles*** of a convex polygon is found by solving $180°(n - 2)$.

     Convex polygons can be divided into triangles.
     In a triangle, $n = 3$, so   In a pentagon, $n = 5$, so
     $(n-2)180° = (3-2)180° = 180°$. $(n-2)180° = (5-2)180° = 3(180°) = 540°$.

   - The ***measure of each interior angle*** of a regular n-gon is found by solving $\dfrac{180°(n-2)}{n}$.

   - The ***exterior angle of a polygon*** is an angle that forms a linear pair with one of the angles of the polygon.

   - Interior angles and their adjacent exterior angles are always ***supplementary***. The sum of the degree measures of the two angles is 180°.

   - The ***remote interior angles of a triangle*** are the two angles nonadjacent to the exterior angle.

   - The ***measure of the exterior angle of a triangle*** equals the sum of the measures of the two remote interior angles.

- The ***Exterior Angle Inequality*** states that an exterior angle of a triangle is greater than either of the remote interior angles.

- The ***Exterior Angle Sum Theorem*** states that if a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360°. The corollary that follows states that the measure of each exterior angle of a regular *n*-gon is $\frac{360°}{n}$.

Example:

If the side of a triangle or a polygon is extended, an angle adjacent to the interior angle is formed. This angle is called an exterior angle of the polygon. In this figure, side $\overline{PR}$ has been extended, ∠*QRP* is the interior angle, and ∠*QRS* is the corresponding exterior angle.
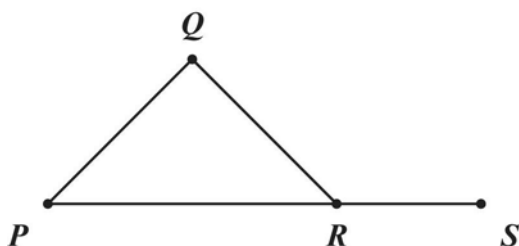
In this figure, side $\overline{QR}$ was extended instead of side $\overline{PR}$. In this case, ∠*QRP* is the interior angle, and ∠*PRT* is the corresponding exterior angle.
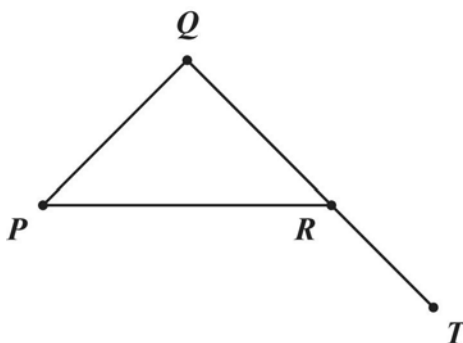
There are always two exterior angles at each vertex of a polygon. When the exterior angle of this triangle is either ∠*QRS* or ∠*PRT*, the remote interior angles are ∠*PQR* and ∠*QPR*.

Notice that the sum of the measures of the remote interior angles is the same for either ∠*QRS* or ∠*PRT* since they both have the same remote interior angles:

$$m\angle QRS = m\angle PQR + m\angle QPR \text{ and } m\angle PRT = m\angle PQR + m\angle QPR$$
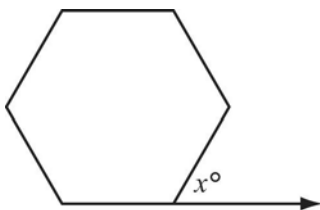
Example:

What is the measure, in degrees, of an interior angle of a regular hexagon?

Solution:

$$\frac{180(n-2)}{n} = \frac{180(6-2)}{6} = \frac{180(4)}{6} = \frac{720}{6} = 120°$$

Example:

Consider this regular hexagon with an exterior angle shown.

What is the value of *x*?

Solution:

$$x = \frac{360}{n} = \frac{360}{6} = 60°$$

The following theorems apply to triangles.

3.   ***Theorem***: If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.

4.   ***Theorem***: If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

5.   ***Theorem***: The sum of the lengths of any two sides of a triangle is greater than the length of the third side (the ***Triangle Inequality Theorem***).

6.   ***Theorem***: If two sides of one triangle are equal to two sides of another triangle but the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is larger than the included angle of the second.

7.   ***Theorem***: If *c* is the measure of the longest side of a triangle, *a* and *b* are the lengths of the other two sides, and $c^2 = a^2 + b^2$, then the triangle is a right triangle (the ***converse of the Pythagorean theorem***).

REVIEW EXAMPLES

1) What is the degree measure of the exterior angle in this figure?
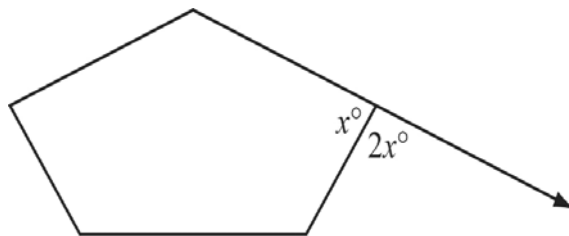
Solution:

In the figure, the number of degrees in the exterior angle of the pentagon is represented by $2x$, and the number of degrees in the adjacent interior angle is represented by x. The exterior angle forms a linear pair with its adjacent interior angle. Therefore,

$$x + 2x = 180$$
$$3x = 180$$

$x = 60$, or $60°$, which is the degree measure of the interior angle
The degree measure of the exterior angle is $2x$ or $2(60) = 120°$.

2) Consider $\triangle CDE$.
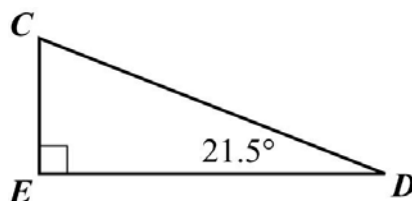
List the sides in order by length from the greatest to the least.

Solution:

You are given $m\angle E = 90°$ and $m\angle D = 21.5°$. First, use the fact that the sum of the measures of the interior angles of a triangle equals 180° to find $m\angle C$.

$$m\angle C = 180 - 90 - 21.5 = 68.5, \text{ or } 68.5°$$

If an angle of a triangle has a measure greater than another angle, then the side opposite the greater angle is longer than the side opposite the smaller angle.

$$m\angle E > m\angle C > m\angle D \text{ ; therefore, } \overline{CD} > \overline{DE} > \overline{EC}$$

EOCT Test Prep Items

1) In $\triangle ECD$, $m\angle E = 136°$, $m\angle C = 17°$, and $m\angle D = 27°$. Which statement must be true?

   A. $\overline{CD} < \overline{DE}$
   B. $\overline{DE} < \overline{CD}$
   C. $\overline{CE} > \overline{CD}$
   D. $\overline{DE} > \overline{CE}$

   [Key: B]

2) Which set could be the lengths of the sides of a triangle?

   A. 15 cm, 18 cm, 26 cm
   B. 16 cm, 16 cm, 32 cm
   C. 17 cm, 20 cm, 40 cm
   D. 18 cm, 22 cm, 42 cm

   [Key: A]

3) The first three angles in a pentagon each have the same measure. The other two angles each measure 10° less than each of the first three angles.

   What is the measure of one of the first three angles in the pentagon?

   A. 102°
   B. 104°
   C. 112°
   **D.** 114°

   [Key: C]

CONGRUENCY

KEY IDEAS

1. The symbol $\cong$ means "is congruent to." If $\triangle ABC \cong \triangle XYZ$,
    then $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{AC} \cong \overline{XZ}$, $\angle A \cong \angle X$, $\angle B \cong \angle Y$, and $\angle C \cong \angle Z$.

The following key ideas are all theorems.

2. ***Theorem:*** If two triangles are congruent, then the corresponding parts of the two
    congruent triangles are congruent.

3. ***SSS Theorem:*** If three sides of one triangle are congruent to three sides of another
    triangle, then the two triangles are congruent.

4. ***SAS Theorem:*** If two sides and the included angle of one triangle are congruent to two
    sides and the included angle of another triangle, then the two triangles are congruent.

5. ***ASA Theorem:*** If two angles and the included side of one triangle are equal to two angles
    and the included side of another triangle, then the two triangles are congruent.
    ASA leads to the ***AAS corollary***. If two angles of a triangle are known, then the third angle
    is also known.

6. ***HL Theorem:*** If the hypotenuse and a leg of one right triangle are congruent to the
    hypotenuse and corresponding leg of another right triangle, then the two triangles are
    congruent.

REVIEW EXAMPLES

1) State the theorem that supports that the two triangles are congruent and write the congruence statement. Explain your reasoning.
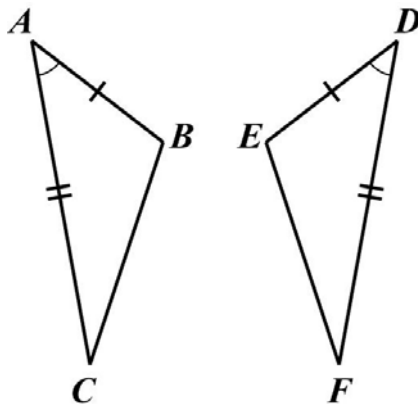
Solution:

The theorem is SAS. The congruence statement is $\triangle ABC \cong \triangle DEF$.

Reasoning:
- The single tick marks on $\overline{AB}$ and $\overline{DE}$ indicate that the **sides** are **congruent**.
- The two tick marks on $\overline{AC}$ and $\overline{DF}$ indicate that these **sides** are **congruent**.
- The angle markings on $\angle A$ and $\angle D$ indicate that these two **angles** are **congruent**.

The drawing shows two congruent sides and the angles between these two sides are congruent (SAS).

2) What theorem supports that the two triangles are congruent? Complete the statement $\overline{AC} \cong$ _____ .
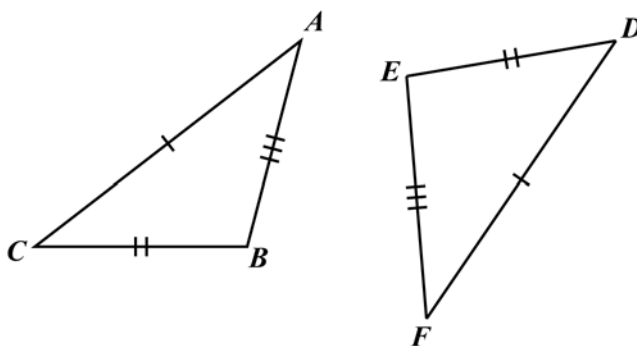
Solution:

By SSS, the two triangles are congruent because the drawing shows three congruent sides. $\triangle ACB \cong \triangle FDE$, so $\overline{AC} \cong \overline{FD}$.

EOCT Test Prep Items

1) **Which set of relationships is sufficient to prove that the triangles in this figure are congruent?**
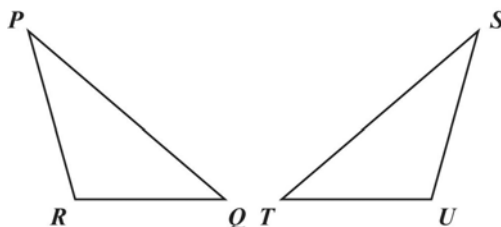
A. $\overline{PR} \cong \overline{SU}, \overline{PQ} \cong \overline{ST}, \angle Q \cong \angle U$
B. $\overline{PQ} \cong \overline{PR}, \overline{ST} \cong \overline{SU}, \overline{RQ} \cong \overline{TU}$
C. $\overline{RQ} \cong \overline{TU}, \angle R \cong \angle U, \angle P \cong \angle S$
D. $\angle P \cong \angle S, \angle R \cong \angle U, \angle Q \cong \angle T$

[Key: C]

2) **Use this diagram of a kite to answer the question.**
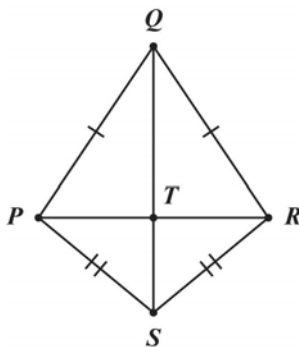
Which statement can be proved by using the HL postulate?

A. $\triangle PQR \cong \triangle PSR$
B. $\triangle PTS \cong \triangle TSR$
C. $\triangle QPS \cong \triangle SRQ$
D. $\triangle QTP \cong \triangle QTR$

[Key: D]

POINTS OF CONCURRENCY IN TRIANGLES

KEY IDEAS

1. Two or more lines that intersect in one point are *concurrent lines*. This intersection point is known as the *point of concurrency*.

2. *Centroid* is the point of concurrency of the medians of a triangle.

    A *median* of a triangle is a segment that joins a vertex of a triangle to the midpoint of the opposite side. The point of concurrency of the medians (the point of intersection) is called the *centroid* of the triangle. The centroid of $\triangle ABC$ is shown in this diagram.
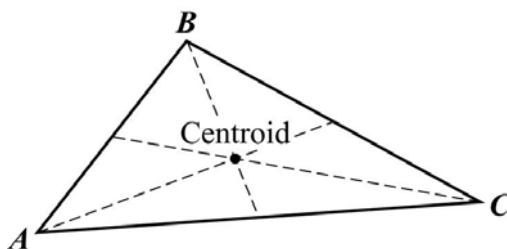
3. *Circumcenter* is the point of concurrency of the perpendicular bisectors of the sides of a triangle. This diagram shows that the three perpendicular bisectors of $\triangle ABC$ are concurrent at a single point.
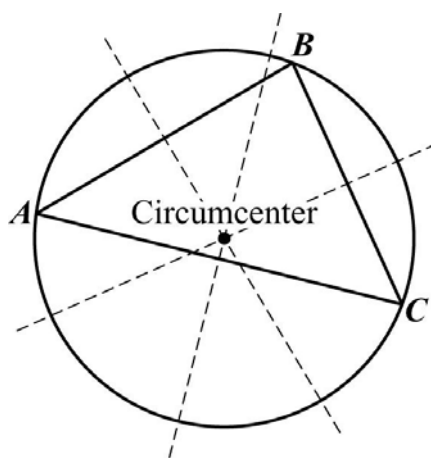
This **point of concurrency** is called the **circumcenter** of the triangle. This point is also the center of the circle **circumscribed** about $\triangle ABC$. Notice that the circle passes through all three vertices of $\triangle ABC$. You can also explain this diagram by stating that $\triangle ABC$ is **inscribed** in the circle.

4.   **Incenter** is the point of concurrency of the bisectors of the angles of a triangle. This diagram shows the angle bisectors of $\triangle ABC$.
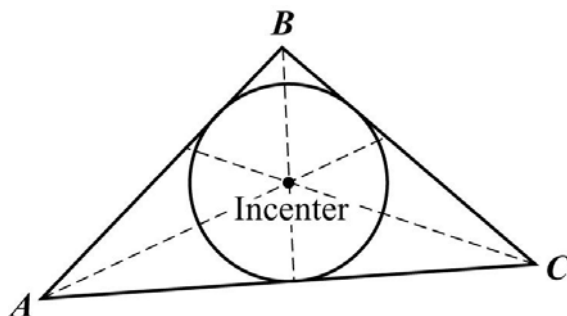
The angle bisectors intersect at a point of concurrency known as the **incenter** of the triangle. It is the center of the circle that can be inscribed in $\triangle ABC$.

5.   **Orthocenter** is the point of concurrency of the altitudes of a triangle.

An **altitude** of a triangle is a perpendicular segment from a vertex of the triangle to the line containing the opposite side. The **point of concurrency** of the lines that contain the altitudes of a triangle (the point of intersection) is called the **orthocenter** of the triangle. This diagram shows the orthocenter of $\triangle ABC$.
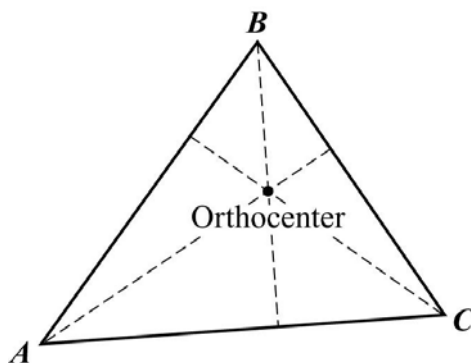
REVIEW EXAMPLES

1) The vertices of △*QRS* are located at *Q*(0, 4), *R*(0, 0), and *S*(6, 0).
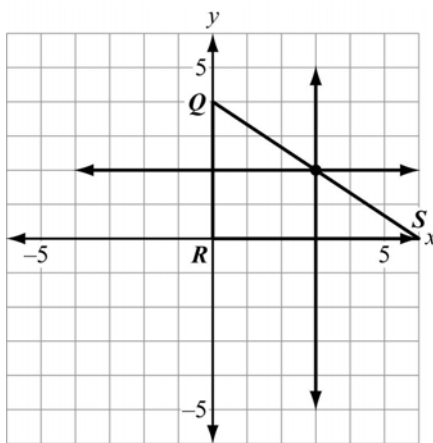
Joe wants to circumscribe a circle about △*QRS*, but he first needs to identify the coordinates of the center of the circle. Use the coordinate grid to identify these coordinates.

Solution:

Two perpendicular bisectors of the sides of △*QRS* are $x = 3$ and $y = 2$. These lines intersect at (3, 2). Therefore, (3, 2) is the location of the center of the circle to be circumscribed about △*QRS*. (This point is called the circumcenter.)

2) A graphic artist plotted a triangular background for a design on the coordinate grid, as shown.
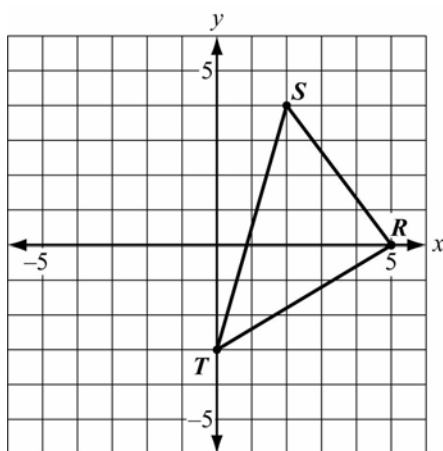
The vertices of △*TRS* are located at *T*(0, −3), *R*(5, 0), and *S*(2, 4). The artist plans to place an icon as the centroid of the triangle. Identify the coordinates of the centroid of △*TRS*.

Solution:

First, find the midpoint of each side of $\triangle TRS$.

- Midpoint of $\overline{TR} = \left(\dfrac{0+5}{2}, \dfrac{-3+0}{2}\right) = (2.5, -1.5)$

- Midpoint of $\overline{RS} = \left(\dfrac{5+2}{2}, \dfrac{0+4}{2}\right) = (3.5, 2)$

- Midpoint of $\overline{ST} = \left(\dfrac{2+0}{2}, \dfrac{4+(-3)}{2}\right) = (1, 0.5)$

Use the midpoints to draw the medians of $\triangle TRS$.
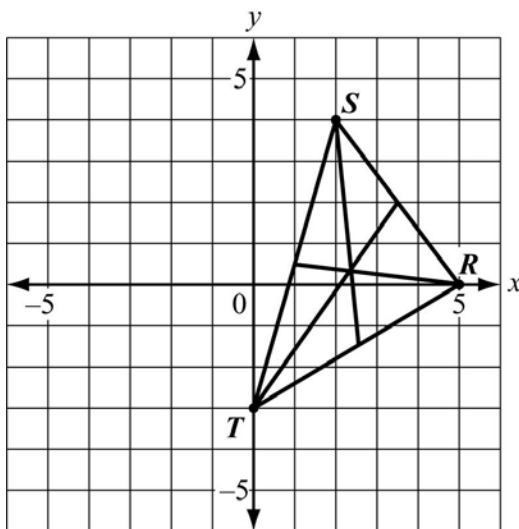
The intersection of the medians is the centroid of the triangle. The location of the centroid is $\left(2\dfrac{1}{3}, \dfrac{1}{3}\right)$.

EOCT Test Prep Items

1) **A student wants to inscribe a circle inside of a triangle. Which of the following should the student construct to locate the incenter of the triangle?**

   A. the medians of the triangles
   B. the altitudes of the triangles
   C. the angle bisectors of the triangle
   D. the perpendicular bisectors of the sides of the triangle

   [Key: C]

2) **Jay constructed a line segment from each vertex that was perpendicular to the line containing the opposite side of a triangle. At what point of concurrency did the lines meet?**

   A. the incenter
   B. the centroid
   C. the orthocenter
   D. the circumcenter

   [Key: C]

PROPERTIES OF AND RELATIONSHIPS AMONG SPECIAL QUADRILATERALS

KEY IDEAS

1. A polygon with four sides is called a *quadrilateral*. The special types of quadrilaterals include parallelogram, rectangle, rhombus, square, trapezoid, and kite. The relationship among these figures is shown in this diagram.
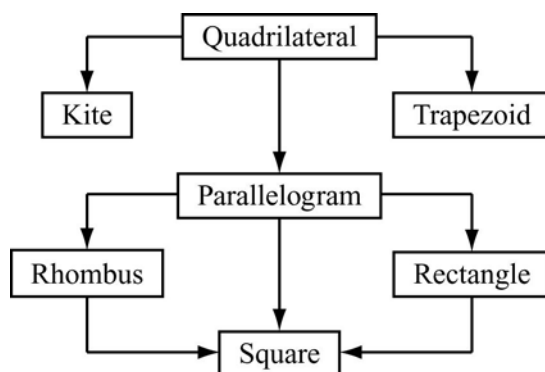
It is important to understand the relationship of these figures and their properties in order to properly classify or identify a figure.

Parallelogram
- opposite sides are parallel
- opposite angles are congruent
- opposite sides are congruent
- diagonals bisect each other
- consecutive angles are supplementary

Rhombus
- has all the properties of a parallelogram
- four sides are equal in length
- diagonals are perpendicular
- diagonals bisect each pair of opposite angles

Rectangle
- has all the properties of a parallelogram
- diagonals are congruent
- contains four right angles

Square
- has all the properties of a parallelogram
- diagonals are congruent and perpendicular
- is a rectangle with all sides congruent
- is a rhombus with four right angles

> **Trapezoid**
> - one pair of opposite sides that are parallel
> - two parallel sides are called bases and the non-parallel sides are the legs
> - isosceles trapezoid has one pair of congruent sides and congruent diagonals

> **Kite**
> A kite is a quadrilateral that has exactly two distinct pairs of adjacent congruent sides

The following key ideas are all theorems.

2. **_Theorem_:** If one pair of opposite sides of a quadrilateral is congruent and parallel, then the quadrilateral is a parallelogram.

3. **_Theorem_:** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

4. **_Theorem_:** If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

5. **_Theorem_:** If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

6. **_Theorem_:** If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

7. **_Theorem_:** If each diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

8. **_Theorem_:** If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

9. **_Theorem_:** If three parallel lines cut off equal segments on one transversal, then they cut off equal segments on every transversal.

REVIEW EXAMPLES

1) The vertices of quadrilateral *PQRS* are plotted at *P*(1, 6), *Q*(6, 7), *R*(7, 2), and *S*(2, 1).
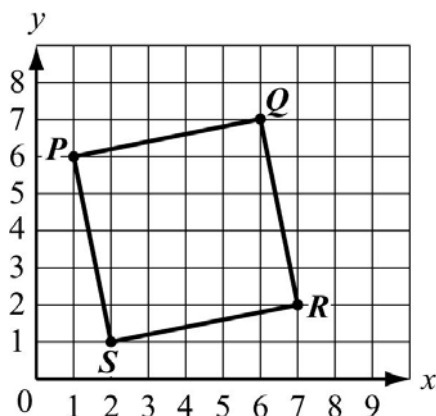
Prove that *PQRS* is a square.

Solution:

One way to prove it is a square is by proving that the two diagonals are congruent and perpendicular.

Use the distance formula $\left(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right)$ to show that $\overline{PR} \cong \overline{QS}$.

- The length of $\overline{PR} = \sqrt{(7-1)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52}$.
- The length of $\overline{QS} = \sqrt{(6-2)^2 + (7-1)^2} = \sqrt{16+36} = \sqrt{52}$.

Since both lines have the same length, the lines are congruent.

Find the slopes of $\overline{PR}$ and $\overline{QS}$ to show that they are perpendicular.

- The slope of $\overline{PR} = \dfrac{6-2}{1-7} = \dfrac{4}{-6} = -\dfrac{2}{3}$.
- The slope of $\overline{QS} = \dfrac{7-1}{6-2} = \dfrac{6}{4} = \dfrac{3}{2}$.

Since the product of the slopes is −1, the lines are perpendicular.

Since both lines are congruent and perpendicular, quadrilateral *PQRS* must be a square.

2) Rectangle *PQRS* is shown in this diagram.
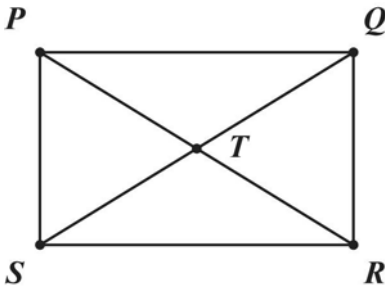
The length of $\overline{SR}$ is 30 centimeters. The length of $\overline{ST}$ is 17 centimeters.
What is the perimeter of the rectangle?

Solution:

The diagonals of a rectangle bisect each other, which means the length of the diagonal is 34 centimeters. The angles of a rectangle are right angles, so use triangle *SQR* and the Pythagorean theorem to find the length of side $\overline{QR}$.

$$a^2 + b^2 = c^2$$
$$a^2 + 30^2 = 34^2$$
$$a^2 + 900 = 1156$$
$$a^2 = 256$$
$$a = 16$$

The perimeter is 2*l* + 2*w* or 2(30) + 2(16) = 60 + 32 = 92, or 92 centimeters.

EOCT Test Prep Items

1) **Which of the following proves that quadrilateral *GHJK* is a parallelogram?**
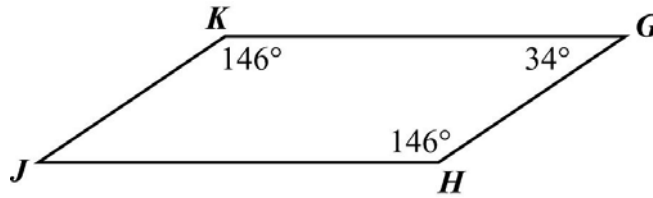
   A. ∠*G* is supplementary to ∠*H* and ∠*K*
   B. ∠*J* is complementary to ∠*G*
   C. *GH* ⊥ *HJ* and *JK* ⊥ *KG*
   D. $\overline{GH} \cong \overline{HJ} \cong \overline{JK} \cong \overline{KG}$

[Key: A]

2) **This diagram shows isosceles trapezoid *QRST*.**
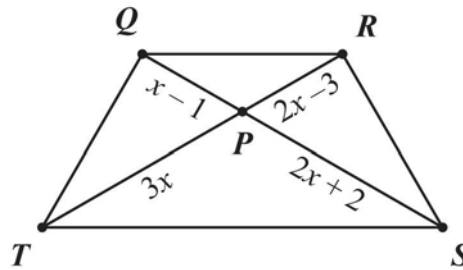
   What is the length, in units, of $\overline{QS}$?

   A. 2
   B. 6
   C. 7
   D. 9

[Key: C]

Unit 4: The Chance of Winning

In this unit, students will calculate probabilities based on angles and area models, compute simple permutations and combinations, calculate and display summary statistics, and calculate expected values. Students will also use simulations and statistics as tools to answer difficult theoretical probability questions.

KEY STANDARDS

MM1D1. Students will determine the number of outcomes related to a given event.
   a. Apply the addition and multiplication principles of counting.
   b. Calculate and use simple permutations and combinations.

MM1D2. Students will use the basic laws of probability.
   a. Find the probabilities of mutually exclusive events.
   b. Find probabilities of dependent events.
   c. Calculate conditional probabilities.
   d. Use expected value to predict outcomes.

MM1D3. Students will relate samples to a population.
   a. Compare summary statistics (mean, median, quartiles, and interquartile range) from one sample data distribution to another sample data distribution in describing center and variability of the data distributions.
   b. Compare the averages of the summary statistics from a large number of samples to the corresponding population parameters.
   c. Understand that a random sample is used to improve the chance of selecting a representative sample.

MM1D4. Students will explore variability of data by determining the mean absolute deviation (the average of the absolute values of the deviations).

COUNTING, PERMUTATIONS, AND COMBINATIONS

KEY IDEAS

1.   The *fundamental counting principle* states that if one event can occur in *m* ways and a second event can occur in *n* ways, then the number of ways that both events can occur is $m \cdot n$ ways. If there is a third event that can occur in *p* ways, then the number of ways that all three events can occur is $m \cdot n \cdot p$ ways. The fundamental counting principle can be extended to any number of events. It can also be used in a situation with or without replacement.

Example:

You are asked to type a pass code in order to gain access to an Internet site. You can use the digits 0 to 9 and the letters A to Z. To find the number of different possible pass codes of 1 digit followed by 3 letters when you can use a letter more than one time, multiply $10 \cdot 26 \cdot 26 \cdot 26 = 175,560$.

There are 10 different digits for the first position and 26 different letters for the second, third, and fourth positions.

Example:

You are asked to type a pass code in order to gain access to an Internet site. You can use the digits 0 to 9 and the letters A to Z. To find the number of different possible pass codes of 1 digit followed by 3 letters when you can NOT use a letter more than once, multiply $10 \cdot 26 \cdot 25 \cdot 24 = 156,000$.

There are 10 different digits for the first position, 26 different letters for the second position, 25 different letters for the third position, and 24 different letters for the fourth position.

2.   *Factorial notation* is indicated by the "!" symbol. The expression *n*! is read "*n* factorial" and means to multiply *n* times $(n - 1)!$ or $n! = n(n - 1)!$. The expression 0! always equals 1.

Example:

The expression 5! tells you to start with 5 and multiply it by 4 and 3 and 2 and 1 or $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

Example:

The expression 4! can be simplified as $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$.

3. A **permutation** is an ordering of a set of objects. When you are concerned with how objects in a set are ordered, you should determine the number of permutations.

4. The number of permutations of n objects taken r at a time can be found using $_nP_r = \dfrac{n!}{(n-r)!}$.

Example:

A swim team has 12 swimmers who can swim in a freestyle event. The swim coach will choose 4 of these swimmers for the freestyle relay race. To determine the number of different swimmer orders that are possible for the coach to choose, use the formula for permutations and let $n = 12$ and $r = 4$.

$$_nP_r = \frac{n!}{(n-r)!} = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 11{,}880$$

There are 11,880 different possible swimmer orders the coach can choose for the relay race.

5. A **combination** is a selection of objects where the order of the objects is not important.

6. The number of combinations of r objects taken from a group of n objects can be found using $_nC_r = \dfrac{n!}{r!(n-r)!}$.

Example:

Using the swim team example from Key Idea **#4**, find the number of possible four-person teams chosen from the eligible swimmers, when the order of the swimmers is NOT important. Use the formula for combinations and let $n = 12$ and $r = 4$.

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_{12}C_4 = \frac{12!}{4!(12-4)!}$$

$$_{12}C_4 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8!}$$

$$_{12}C_4 = 495$$

There are 495 possible teams of four.

Example:

Heather is buying a 2-topping pizza. She has 6 different toppings from which to choose.

To find the number of different combinations of 2 toppings that are possible, use

$$_nC_r = \frac{n!}{r!(n-r)!}.$$

$$_nC_r = \frac{n!}{r!(n-r)!}$$
$$_6C_2 = \frac{6!}{2!(6-2)!}$$
$$_6C_2 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{(2 \cdot 1)(4 \cdot 3 \cdot 2 \cdot 1)}$$
$$_6C_2 = 15$$

There are 15 different possible combinations of 2 toppings.

7. When finding the number of ways that one event or another event can occur, you need to find the number of ways each event can occur and add.

Example:

Eric has 3 sandwiches, 2 salads, and 4 drinks. He will choose a sandwich, a salad, and a drink, or he will choose a salad and a drink. Find the number of combinations possible.

"He will choose a sandwich, a salad, AND a drink"   "OR"   "salad AND a drink"

Multiply   ADD   Multiply
3 sandwiches × 2 salads × 4 drinks   +   2 salads × 4 drinks
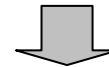
(3 × 2 × 4) + (2 × 4) = 24 + 8 = 32 combinations possible

REVIEW EXAMPLES

1) Drake can choose from 31 flavors of ice cream. He wants to get a bowl with four scoops of ice cream. Each of the four scoops of ice cream will be a different flavor.

   How many different bowls of four scoops of ice cream are possible?

Solution:

   The event of selecting a flavor of ice cream after the first selection is dependent on the previous event, since each scoop of ice cream will be a different flavor.

   - Scoop 1 has 31 options.
   - Scoop 2 has $31 - 1 = 30$ options.
   - Scoop 3 has $30 - 1 = 29$ options.
   - Scoop 4 has $29 - 1 = 28$ options.

   Multiply $31 \times 30 \times 29 \times 28 = 755{,}160$ different bowls of four scoops of ice cream.

2) From a group of 5 nutritionists and 7 nurses, Elyse must select a committee consisting of 2 nutritionists and 3 nurses. In how many ways can she do this if one particular nurse must be on the committee?

Solution:

   In this scenario, consider that there is more than one event.
   First event: choose the nutritionists. Let $n = 5$ and $r = 2$.

   $$_nC_r = \frac{n!}{r!(n-r)!} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!(3)!} = \frac{5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{(2 \cdot 1) \cdot \cancel{(3 \cdot 2 \cdot 1)}} = \frac{20}{2} = 10$$

   Second event: choose the nurses. The problem states that one particular nurse must be on the committee, so let $n = 7 - 1 = 6$, and $r = 3 - 1 = 2$.

   $$_nC_r = \frac{n!}{r!(n-r)!} = \frac{6!}{2!(6-2)!} = \frac{6!}{2!(4)!} = \frac{6 \cdot 5 \cdot \cancel{4 \cdot 3 \cdot 2 \cdot 1}}{(2 \cdot 1) \cdot \cancel{(4 \cdot 3 \cdot 2 \cdot 1)}} = \frac{30}{2} = 15$$

   To find the possible committees, multiply the combinations $10 \times 15 = 150$.

   Elyse has 150 possible ways to form the committee.

EOCT Test Prep Items

1) **There are five points in a plane, but no three points are collinear. How many different straight lines that pass through two of the points are possible?**

   A. 2
   B. 10
   C. 15
   D. 20

   [Key: B]

2) **Danny has 3 identical color cubes. Each of the 6 faces on the color cubes is a different color. He also has 2 fair coins.**

   What is the total number of possible outcomes if Danny rolls all three cubes OR flips both coins?

   A. 22
   B. 144
   C. 220
   D. 864

   [Key: C]

3) **There are 14 students in a mathematics competition. Each student will earn points during the competition. The student with the greatest number of points will be the first place winner, and the student with the second greatest number of points will be the second place winner.**

   How many different ways can the 14 students finish in first place and second place?

   A. 27
   B. 91
   C. 182
   D. 196

   [Key: C]

PROBABILITY

KEY IDEAS

1. The *probability* of an event is the ratio of the number of outcomes in the event to the total number of outcomes in the sample space. It is the likelihood that an event will occur.

$$P(E) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes in the sample space}}$$

   The probability of an event occurring is represented by a number between 0 and 1. A probability of 0 means that the occurrence of the event is NOT possible. A probability of 1 means that the occurrence of the event is absolutely certain.

2. Two events that have no outcomes in common are called *mutually exclusive* events. They are two events that cannot occur at the same time.

3. Events are *random events* when individual outcomes are uncertain. However, there may be a regular distribution of outcomes in a large number of repetitions. An example of this is flipping a fair coin. If you flip it enough times it will land with heads facing up about 50% of the time and tails facing up about 50% of the time. However, the outcome of a single flip of the coin is uncertain.

4. The *addition rule for mutually exclusive events* is shown by the formula
    $P(A \text{ or } B) = P(A) + P(B)$.

    Example:

    The sample space for rolling a number cube is {1, 2, 3, 4, 5, 6}, with each number representing the top face of the cube. Only one number can be on the top face at a time.

    To find the probability of a number cube landing with 2 facing up or landing with 5 facing up (mutually exclusive events), find the probability of the cube landing on 2, and then find the probability of the cube landing on 5. Then find the sum of the two probabilities of the two events: $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

5. The *addition rule for sets that are NOT mutually exclusive* is shown by the formula
    $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Example:

For a number cube with faces numbered from 1 to 6, the events of "rolling a number cube and having it land with an even number on the top face" and "rolling a number cube and having it land with a number greater than two on the top face" are not mutually exclusive.

Let event *A* represent rolling the number cube and having it land with an even number on the top face. Let event *B* represent rolling the number cube and having it land with a number greater than two on the top face.

One way to find the probability of *A* or *B* is to use the formula
$P(A \text{ or } B) = P(\text{even number}) + P(\text{number greater than two}) - P(\text{even number and number greater than two}) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$.

Another way is to examine the sample space $\{1, 2, 3, 4, 5, 6\}$ and count the number of outcomes that are even numbers or are numbers that are greater than two. Those numbers are 2, 3, 4, 5, and 6. That is 5 out of the 6 possible outcomes. The probability would be $\frac{5}{6}$.

6. Two events are ***independent events*** if the outcome of the first event does not affect the probability of the second event.

7. Two events are ***dependent events*** when the outcome of the first event affects the probability of the second event.

Example:

Suppose 2 cards are drawn from a standard deck of 52 cards *without replacement*. The probability that both cards are clubs would be $\frac{13}{52} \cdot \frac{12}{51}$ or $\frac{1}{17}$. If a club was drawn first, then there would be only 12 clubs left out of 51 cards left, since the first club was not returned to the deck. The probability of drawing a club on the second draw is different than the probability of drawing a club on the first draw because the events are dependent.

8. ***Conditional probability*** is a type of dependent probability. Given two events, conditional probability is the probability of event *B*, given that event *A* has occurred. Conditional probability is denoted as $P(B|A)$ and can be found using $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)}$.

Example:

You toss a coin and roll a number cube that has faces numbered from 1 to 6. Find the probability that the number cube will land with an odd number facing up, given that tails is facing up on the coin.

Solution:

There are two events:

Event *A*: Tossing a coin   Event *B*: Rolling a fair number cube

$$P(A) = \frac{1}{2} \leftarrow \frac{\text{Landed on tails}}{\text{Two outcomes possible}} \qquad P(B) = \frac{3}{6} \leftarrow \frac{\text{3 odd numbers on a number cube}}{\text{6 total numbers on a number cube}}$$

Find $P(B|A) = \dfrac{P(A \text{ and } B)}{P(A)} = \dfrac{\frac{1}{2} \cdot \frac{3}{6}}{\frac{1}{2}} = \dfrac{\frac{1}{4}}{\frac{1}{2}} = \dfrac{1}{2}$

The probability can also be found by making a list of the sample space. Tails is facing up on the coin, so the sample space is:

$$(T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)$$

There are 3 favorable outcomes, (T,1), (T, 3), and (T,5), out of the 6 possible.

So, $P(B|A) = \dfrac{3}{6} = \dfrac{1}{2}$.

9. *Expected value* is the sum of the probability of each possible outcome of an event multiplied by the value of each outcome.

Expected value = $p_1 o_1 + p_2 o_2 + p_3 o_3 + \ldots + p_n o_n$, where *p* is the probability of each outcome and *o* is the value of each outcome.

This value represents the long-term average of the probability of each outcome weighted by the payoff for that outcome. Keep in mind that the expected value may not be an actual outcome, as it represents an *average* value, which could lie between two actual outcome values.

Example:

Suppose you are playing a game with a number cube that has its six faces numbered 1, 1, 1, 1, 2, and 3. The expected value for rolling the cube would be 1.5.

$$E(\text{number cube}) = \frac{1}{6}(1) + \frac{1}{6}(1) + \frac{1}{6}(1) + \frac{1}{6}(1) + \frac{1}{6}(2) + \frac{1}{6}(3)$$
$$= \frac{1+1+1+1+2+3}{6}$$
$$= \frac{9}{6}$$
$$= 1.5$$

Note: The expected value, 1.5, is a value that is impossible to roll, yet represents the average value of a roll of the number cube in the example.

REVIEW EXAMPLES

1) Two number cubes are rolled that each have faces numbered from 1 to 6. What is the probability that the sum of the numbers on the top face of each cube is 4 or 5?

Solution:

This diagram shows the sample space for a roll of two cubes organized such that each row shows the possible outcomes for a specific sum.
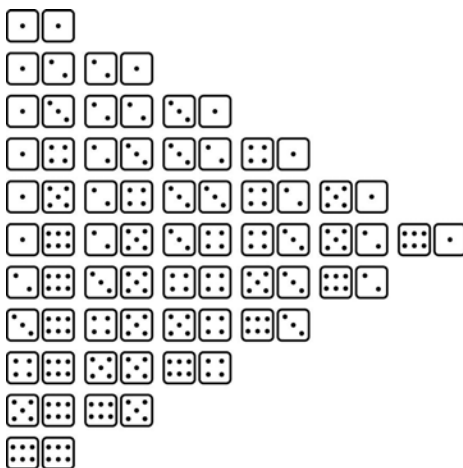
There are 36 possible outcomes when two cubes are rolled.
There are 3 possible outcomes with a sum of 4.
There are 4 possible outcomes with a sum of 5.
Thus, there are 7 possible outcomes with a sum of 4 or 5 out of 36 possible outcomes, and the probability is $\frac{7}{36}$.

Check:

$$P(A \text{ or } B) = P(A) + P(B) = \frac{3}{36} + \frac{4}{36} = \frac{7}{36}$$

2) City consultants conducted a survey of 100 people to determine the community interest in constructing a new fire station. The results are shown in this table.

	Males	Females
Supports	27	25
Opposes	18	13
No Opinion	5	12

a. Find the probability that a randomly selected survey participant supports the construction of a new fire station or has no opinion.
b. Find the probability that a randomly selected survey participant does NOT support the construction of a new fire station.
c. Find the probability that a randomly selected survey participant is female or opposes the construction of a new fire station.

Solution:

a. The number of people who support the fire station is $27 + 25 = 52$.
The number of people who have no opinion is $5 + 12 = 17$.

Supporting the fire station or having no opinion are two mutually exclusive events, as a person cannot answer the survey question with both responses at the same time.

$$P(\text{supporting } or \text{ no opinion}) = P(\text{supporting}) + P(\text{no opinion})$$
$$= \frac{52}{100} + \frac{17}{100}$$
$$= \frac{69}{100}$$

The probability that a randomly selected survey participant supports the construction of a new fire station or has no opinion is $\frac{69}{100}$ or 69%.

b. The probability that a randomly selected survey participant does NOT support the construction of a new fire station is the *complement* of part a.

Recall that a ***complement of an event*** E is $P(\text{not } E) = 1 - P(E)$. So, to find the probability that a survey participant does NOT support the construction of a new fire station, subtract:

$100\% - 69\% = 31\%$ or $\frac{31}{100}$.

c. The probability that a randomly selected survey participant is female is

$\frac{25+13+12}{100} = \frac{50}{100}$. The probability that a randomly selected survey participant opposes

the construction of a new fire station is $\frac{18+13}{100} = \frac{31}{100}$. The probability that a randomly

selected survey participant is female AND against the construction of a new fire station is

$\frac{13}{100}$. Note that this group of 13 participants has been counted both in the "female" group

and the "opposes the construction of a new fire station" group; this is why it must be

subtracted from the total. The probability that a randomly selected survey participant is

female or is against the construction of a new fire station is $\frac{50}{100} + \frac{31}{100} - \frac{13}{100} = \frac{68}{100}$ or 68%.

3) Keira is playing a game at a school carnival. She pays $1 to play the game. In the game, there are eight identical small doors. Behind six of the doors there is nothing. Behind one of the doors there is a $1 coupon she can use to play the game again. Behind one of the doors there is a prize worth $5. The coupon and prize are assigned randomly to doors each time a person plays the game.

   a. What is the expected value of playing the game one time?
   b. What is the net value of playing the game one time?

Solution:

   a. There are eight doors, each of which is an equally likely outcome for the game.

      - The value of each of six of the doors is $0.
      - The value of one of the doors is $1.
      - The value of one of the doors is $5.

      The expected value of playing the game one time is $0.75.

      $\frac{6}{8}(0) + \frac{1}{8}(1) + \frac{1}{8}(5) = \frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4}$ of a dollar, or $0.75.

   b. The net value of playing the game is found by taking the expected value of playing the game and subtracting from it the cost of playing the game.

      $\$0.75 - \$1.00 = -\$0.25$

      The net value of playing the game one time is −$0.25, or a loss of $0.25. Note that if Keira plays the game 100 times she can expect to lose a total of $100 \times \$0.25 = \$25$.

EOCT Test Prep Items

1) **A teacher has 9 red crayons, 4 blue crayons, 7 purple crayons, and 5 black crayons in a basket. A student reaches into the basket and randomly selects a crayon. What is the probability that the crayon will be either blue or black?**

   A. $\dfrac{4}{16}$

   B. $\dfrac{9}{25}$

   C. $\dfrac{13}{25}$

   D. $\dfrac{9}{16}$

   [Key: B]

2) **There are 6 red apples, 4 yellow apples, and 2 green apples in a bucket. Maria will choose two apples at random without replacement.**

   What is the probability that Maria will choose a red apple and then a green apple?

   A. $\dfrac{5}{121}$

   B. $\dfrac{6}{121}$

   C. $\dfrac{1}{11}$

   D. $\dfrac{1}{12}$

   [Key: C]

3) **This table shows the probability of each possible sum when two cubes with faces numbered 1 through 6 are rolled and the numbers showing on each face are added.**

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Seth is playing a game in which he gets 10 points when the sum is a perfect square. He gets 5 points if the sum is a prime number. He gets 0 points if the sum is a number that is neither prime nor a perfect square.

What is the expected value, to the nearest 0.1, for one roll of the two number cubes?

A. 4.0 points
B. 4.1 points
C. 4.3 points
D. 4.6 points

[Key: A]

SUMMARY STATISTICS

KEY IDEAS

1. The *measures of central tendency* are mean, median, and mode.

    - The *mean* of a data set can be determined by dividing the sum of the data values by the total number of data values.
    - The *median* of a data set is the middle value of the data set, or the average of the two middle values of a data set with an even number of values when the data is placed in order.
    - The *mode* of a data set is the most frequent value in the data set.

2. The *first quartile* or the *lower quartile*, Q_1, is the median of the lower half of a data set.

    Example:

    Ray's scores on his first 7 mathematics tests were 70, 85, 78, 90, 84, 82, and 83. To find the first quartile of his scores, put them in order and find the median of the lower half.

    $$70, 78, 82, 83, 84, 85, 90$$

    The first quartile is 78.

3. The *third quartile* or the *upper quartile*, Q_3, is the median of the upper half of a data set.

    Example:

    To find the third quartile of Ray's scores from the example in Key Idea **#2**, put the scores in order and find the median of the upper half. The third quartile is 85.

4. The *interquartile range* of a data set is the difference between the quartiles or $Q_3 - Q_1$.

    Example:

    To find the interquartile range of Ray's scores from the example in Key Idea **#2**, subtract the first quartile from the third quartile. The interquartile range of Ray's scores is $85 - 78 = 7$.

5. A *population* is a group of people, animals, or objects and a *sample* is part of the population. The general goal of all sampling methods is to obtain a sample that is representative of the target population.

6.   One way to obtain a representative sample of the population is to use a random sample. A
      random sample is one where every person in the population from which the sample is
      drawn has an equal chance of being included.

      Example:

      A teacher wants to randomly select a student. To do this, the names of each student are
      written on a piece of paper and placed in a paper bag. The teacher then draws a piece of
      paper from the bag without looking at the name. This method ensures that each student
      has an equal chance of being selected.

      If the teacher selects a student from among those students who raised their hands, this is
      NOT random. Not every student has an equal chance of being selected.

REVIEW EXAMPLES

1) John surveys every fifth person leaving a pet supply store. Of those surveyed, $\frac{3}{4}$ support the
    city manager's proposition to tear down the old library structures and replace the area with
    the construction of a new pet park. John plans to write a letter to the editor of the local
    newspaper about the proposal for the new pet park stating that there is tremendous support
    from the citizens of the town for constructing a new pet park.
    a. Can the conclusion John formed be accurately supported?
    b. Suggest another plan for obtaining a good sample population.

Solution:

    a. The population that John studied is likely to have a very favorable opinion about pets, as
        they are shopping in a pet supply store. It is likely that this population of people would be
        strong supporters for a new pet park and would therefore cause the findings to be biased.
        To conclude that there is tremendous support from the citizens of the town may not be
        true because John only sampled the population of people who shopped at a pet store. The
        population studied was not representative of the people in the entire town.

    b. To obtain a representative sample from the population of people in the town, select every
        fiftieth person from the data bank of registered voters. This population would represent
        the people who will vote on the issue, and likely a good mix of people who are fond of
        pets, people who are fond of libraries, people who are fond of architecture, or none of the
        above.

2) Warren and Mason each get paid a bonus at the end of each month. This table shows their bonuses for the first five months of the year.

Month	Warren's Bonus	Mason's Bonus
January	$250	$250
February	$250	$320
March	$270	$310
April	$240	$300
May	$260	$260

   a. Who had the greatest median bonus? What is the difference in the median of Warren's bonuses and the median of Mason's bonuses?

   b. What is the difference in the interquartile range for Warren's bonus and Mason's bonus?

Solution:

   a. Warren's median bonus is $250. Put Warren's bonuses in order and find the middle one.

$$\$240, \$250, \textbf{\$250}, \$260, \$270$$

Mason's median bonus is $300. Put Mason's bonuses in order and find the middle one.

$$\$250, \$260, \textbf{\$300}, \$310, \$320$$

Mason had the greatest median bonus. The difference in the median of the bonuses is

$$300 - 250 = 50 \text{ or } \$50.$$

   b. The lower quartile for Warren is $245 and the upper quartile is $265. The interquartile range is the difference in these or $20.

The lower quartile for Mason is $255 and the upper quartile is $315. The interquartile range is the difference in these or $60.

The difference in Warren's interquartile range and Mason's interquartile range is

$$60 - 20 = 40 \text{ or } \$40.$$

3) Jessica is a student at Adams High School. These histograms give information about the number of hours of community service completed by each of the students in Jessica's homeroom and by each of the students in the ninth-grade class at her school.
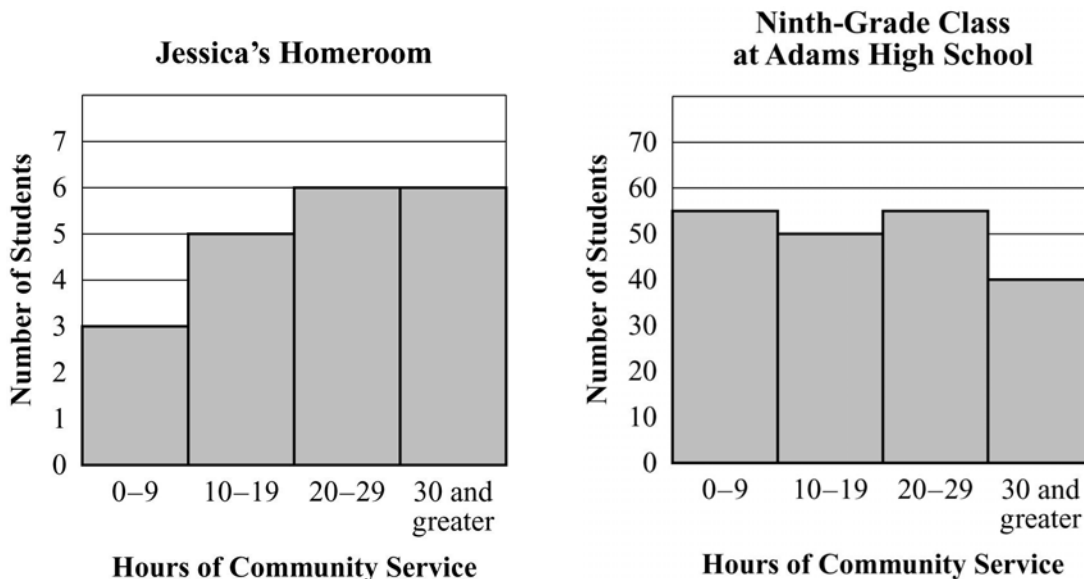
a. Compare the lower quartiles of the data in the histograms.
b. Compare the upper quartiles of the data in the histograms.
c. Compare the medians of the data in the histograms.

Solution:

a. You can add the number of students in each bar to find there are 20 students in Jessica's homeroom. The lower quartile is the median of the first half of the data. That would be found within the second bar, which represents 10-19 hours.

   You can add the number of students in each bar to find out there are 200 students in the ninth-grade class. The lower quartile for this group is found within the first bar, which represents 0-9 hours.

   The lower quartile of the number of community service hours completed by each student in Jessica's homeroom is greater than the lower quartile of community service hours completed by each student in the ninth-grade class.

b. The upper quartile is the median of the second half of the data. For Jessica's homeroom, that would be found within the fourth bar, which represents 30 and greater hours.

   For the ninth-grade class, the upper quartile is found within the third bar, which represents 20-29 hours.

The upper quartile of the number of community service hours completed by each student in Jessica's homeroom is greater than the upper quartile of community service hours completed by each student in the ninth-grade class.

c. The median is the middle number in a data set that is written in order from least to greatest or greatest to least. The median for Jessica's homeroom is found within the third bar, which represents 20-29 hours.

The median for the ninth-grade class is found within the second bar, which represents 10-19 hours.

The median of the number of community service hours completed by each student in Jessica's homeroom is greater than the median of community service hours completed by each student in the ninth-grade class.

EOCT Test Prep Items

1) **This table shows the average high temperature, in ºF, recorded in Atlanta, GA, and Austin, TX, over a six-day period.**

Day	1	2	3	4	5	6
Temperature in °F in Atlanta, GA	85	88	83	79	81	85
Temperature in °F in Austin, TX	80	86	82	80	93	89

Which conclusion can be drawn from the data?

A. The median temperature over the six-day period was the same for both cities, but the interquartile range is greater for Austin than Atlanta.

B. The mean temperature of Atlanta was higher than the mean temperature of Austin, and the interquartile range is greater for Atlanta than Austin.

C. The mean temperature of Austin was higher than the mean temperature of Atlanta, but the median temperature of Austin was lower.

D. The mean and median temperatures of Atlanta were higher than the mean and median temperatures of Austin.

[Key: A]

2) **A school was having a canned food drive for a local food bank. A teacher determined the median number of cans collected per class and the interquartile ranges of the number of cans collected per class for the juniors and for the seniors.**

- **The juniors collected a median number of cans per class of 35, and the interquartile range was 10.**
- **The seniors collected a median number of cans per class of 40, and the interquartile range was 8.**
- **Both the juniors and the seniors had the same third quartile number of cans collected.**

Which range includes only the numbers that could be the third quartile number of cans collected for both classes?

A. 25 to 45
B. 25 to 48
C. 32 to 48
D. 40 to 45

[Key: D]

MEAN ABSOLUTE DEVIATION

KEY IDEAS

1. The ***mean absolute deviation*** is a measure of spread (or variability). It is the mean amount by which the values in a data set differ, or vary, from the mean.

2. The mean absolute deviation of a set of data can be determined by following these steps:

   - Calculate the mean of the data set.
   - Find the absolute values of the differences of each data point and the mean.
   - Find the sum of the absolute values.
   - Divide the sum by the total number of values in the data set.

   Example:

   The mean of the data set, 6, 6, 9, 4, 5 is 6. To determine the mean absolute deviation, find the absolute values of the differences of each data point and 6 (the mean).

   $$|6-6|=0; \ |6-6|=0; \ |9-6|=3; \ |4-6|=2; \ |5-6|=1$$

   Find the sum: $0 + 0 + 3 + 2 + 1 = 6$

   Divide 6 by the total number of data values: $6 \div 5 = 1.2$

   The mean absolute deviation is 1.2.

REVIEW EXAMPLES

1) What is the mean absolute deviation of the following data set?

   25, 57, 44, 34

Solution:

First find the mean of the data set: $\dfrac{25+57+44+34}{4} = 40$

Then find the absolute values of the difference between each data point and 40.
$$|40-25|+|40-57|+|40-44|+|40-34|=42$$

Divide the sum of 42 by the number of data points: $42 \div 4 = 10.5$

The mean absolute deviation of the set is 10.5.

2) Emma and Sara play 5 games on a handheld video game and record their scores in this table.

Game Score

Game	Emma	Sara
1	30	15
2	20	25
3	10	20
4	20	25
5	20	15

Which girl had the greater mean deviation for her scores?

Solution:

Emma's mean score is $\dfrac{30+20+10+20+20}{5}=20$

Find the absolute values of the differences between each data point and 20.

$$|30-20|=10; \ |20-20|=0; \ |10-20|=10; \ |20-20|=0; \ |20-20|=0$$

Find the sum: $10+0+10+0+0=20$

Divide the sum of 20 by the total number of data values: $20 \div 5 = 4$

Sara's mean score is $\dfrac{15+25+20+25+15}{5}=20$

Find the absolute values of the differences of each data point and 20.

$$|15-20|=5; \ |25-20|=5; \ |20-20|=0; \ |25-20|=5; \ |15-20|=5$$

Find the sum: $5+5+0+5+5=20$

Divide the sum of 20 by the total number of data values: $20 \div 5 = 4$

The absolute mean deviation is 4 for both Emma and Sara. Neither one had a greater mean absolute deviation.

EOCT Test Prep Items

1) **This table shows the scores of four students on their first four mathematics quizzes.**

Student	Quiz Scores	Mean Score
Anna	80, 90, 75, 75	80
Jim	100, 70, 80, 70	80
Stacy	92, 78, 68, 82	80
Ted	71, 84, 84, 81	80

Which student had the least mean absolute deviation on the quiz scores?

A. Anna
B. Jim
C. Stacy
D. Ted

[Key: D]

2) **The heights, in inches, of five girls in an exercise class were 66, 64, 68, 70, and 65. A sixth girl joined the class. The mean height of the six girls in the class was 66 inches and the mean absolute deviation was 2 inches. What was the height of the sixth girl who joined the class?**

A. 63 inches
B. 64 inches
C. 65 inches
D. 66 inches

[Key: A]

Unit 5: Algebra in Context

The focus of this unit is on the development of students' abilities to solve simple quadratic, rational, and radical equations using a variety of methods. Students extend and apply the skills and understandings of Units 1 and 2 through further investigation of quadratic, rational, and radical functions. The even and odd symmetry of graphs will be explored, as well as the intersections of graphs as solutions to equations.

KEY STANDARDS

MM1A1. Students will explore and interpret the characteristics of functions, using graphs, tables, and simple algebraic techniques.
    c. Graph transformations of basic functions including vertical shifts, stretches, and shrinks, as well as reflections across the *x*- and *y*-axes.
    d. Investigate and explain characteristics of a function: domain, range, zeros, intercepts, intervals of increase and decrease, maximum and minimum values, and end behavior.
    h. Determine graphically and algebraically whether a function has symmetry and whether it is even, odd, or neither.
    i. Understand that any equation in *x* can be interpreted as the equation $f(x) = g(x)$, and interpret the solutions of the equation as the *x*-value(s) of the intersection point(s) of the graphs of $y = f(x)$ and $y = g(x)$.

MM1A2. Students will simplify and operate with radical expressions, polynomials, and rational expressions.
    a. Simplify algebraic and numeric expressions involving square root.
    b. Perform operations with square roots.
    c. Add, subtract, multiply, and divide polynomials.
    e. Add, subtract, multiply, and divide with rational expressions.
    f. Factor expressions by greatest common factor, grouping, trial and error, and special products limited to the formulas below.

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + y)(x - y) = x^2 - y^2$$
$$(x + a)(x + b) = x^2 + (a + b)x + ab$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

MM1A3. Students will solve simple equations.
    a. Solve quadratic equations in the form $ax^2 + bx + c = 0$, where $a = 1$, by using factorization and finding square roots where applicable.
    b. Solve equations involving radicals such as $\sqrt{x} + b = c$, using algebraic techniques.
    c. Use a variety of techniques, including technology, tables, and graphs to solve equations resulting from the investigation of $x^2 + bx + c = 0$.
    d. Solve simple rational equations that result in linear equations or quadratic equations with leading coefficient of 1.

QUADRATIC EQUATIONS

KEY IDEAS

1.   A *quadratic equation* is an equation that can be written in the standard form
      $ax^2 + bx + c = 0$.

2.   The *Addition Property of Equality* allows us to get an equivalent equation by adding the
      same expression to both sides of the equation. The *Multiplicative Property of Equality*
      allows us to get an equivalent equation by multiplying both sides of the equation by the
      same number as long as the number used is not 0.

3.   The *Zero Product Property* states that for real numbers x and y, if $xy = 0$, then $x = 0$ or
      $y = 0$, or x and y both equal 0.

4.   One way to solve a quadratic equation in the form $x^2 + bx + c = 0$ is by factoring and
      applying the special product $(x + a)(x + b) = x^2 + (a + b)x + ab$.

Example:

Solve the equation: $x^2 + 5x + 6 = 0$.

Solution:

$(x + 2)(x + 3) = 0$		Factor.
$x + 2 = 0$	$x + 3 = 0$	Use the Zero Product Property to set each factor equal to 0.
$x + 2 - 2 = 0 - 2$	$x + 3 - 3 = 0 - 3$	Use the properties of equality to solve each equation.
$x = -2$ or	$x = -3$	There are two solutions.

The answer can be checked by substituting it into the original equation and simplifying.
The statement will be true if the answer is correct.

Check x = –2	*Check x = –3*
$(-2)^2 + 5(-2) + 6 = 0$	$(-3)^2 + 5(-3) + 6 = 0$
$4 - 10 + 6 = 0$	$9 - 15 + 6 = 0$
$0 = 0$	$0 = 0$
This is true, so -2 is a solution.	This is true, so -3 is a solution.

5.   One way to solve a quadratic equation in the form $x^2 - c = 0$, where the value of c is a perfect square, is to use the ***difference of squares*** identity.

    Example:

$$x^2 - 36 = 0$$
$$(x-6)(x+6) = 0$$

$x - 6 = 0$	$x + 6 = 0$
$x - 6 + 6 = 0 + 6$	$x + 6 - 6 = 0 - 6$
$x = 6$ or	$x = -6$

6.   A quadratic equation in the form $x^2 - c = 0$ always has two solutions for $c > 0$, $\sqrt{c}$ and $-\sqrt{c}$. This is true whether the value of c is a perfect square or not.

    Example:

$$x^2 - 7 = 0$$
$$x = \sqrt{7} \text{ or } x = -\sqrt{7}$$

7.   There is an important distinction between solving an equation and solving an applied problem modeled by an equation. The situation that gave rise to the equation may include restrictions on the solution to the applied problem that eliminate certain solutions to the equation.

REVIEW EXAMPLES

1) Carrie has a rectangular butterfly garden that is 12 feet long by 8 feet wide. She wants to put a sidewalk along two sides of the garden, as shown by the shaded area of this diagram.
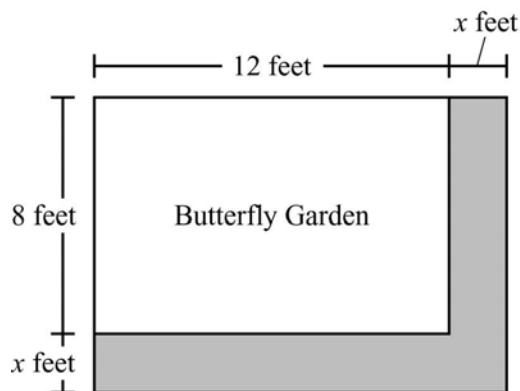
    Carrie has enough concrete for the sidewalk to cover 44 square feet. What is the maximum width she can make her sidewalk?

Solution:

One way to do this is to divide the shaded area into two rectangles. Represent the area of each rectangle algebraically, add them together, and set them equal to 44, the number of square feet Carrie can cover with concrete.

$12x + x(8 + x) = 44$

$12x + 8x + x^2 = 44$ Simplify.

$x^2 + 20x = 44$ Simplify.

$x^2 + 20x - 44 = 0$ Use algebraic rules for creating equivalent equations
         to obtain an equivalent equation with an expression in
         terms of x on one side of the equation and 0 on the other.

$(x - 2)(x + 22) = 0$ Factor.

$x - 2 = 0$ $x + 22 = 0$ Set each factor equal to 0.

$x - 2 + 2 = 0 + 2$ $x + 22 - 22 = 0 - 22$ Solve each equation.

     $x = 2$ or $x = -22$

Since the question asks for the width of a sidewalk, the first solution of 2 feet makes sense. The second solution of –22 feet does not make sense, so eliminate it. Therefore, the maximum width Carrie can make her sidewalk is 2 feet.

2) A right triangle has one leg that is 9 centimeters long. Its hypotenuse is 10 centimeters long. What is the length, in centimeters, of the other leg?

Solution:

Represent the unknown side with a variable, such as a. Then use the Pythagorean theorem to set up a relationship.

$a^2 + 9^2 = 10^2$

$a^2 + 81 = 100$ Simplify.

$a^2 + 81 - 100 = 100 - 100$ Subtract 100 from each side.

$a^2 - 19 = 0$ Simplify.

$a = \sqrt{19}$ or $a = -\sqrt{19}$

The length of the other leg of the triangle is $\sqrt{19}$ centimeters. Since this solution represents a length, take the square root to get approximately 4.4 centimeters for the answer.

Note that $-\sqrt{19}$ is eliminated as an answer because a length cannot be negative.

EOCT Test Prep Items

1) **A squirrel in a tree dropped an acorn 48 feet to the ground. The number of seconds, t, it took the acorn to reach the ground is modeled by this equation.**

$$-16t^2 + 48 = 0$$

How many seconds did it take the acorn to reach the ground?

A. $\sqrt{3}$
B. 3
C. $\sqrt{32}$
D. 32

[Key: A]

2) **An art teacher painted a rectangular picture on the art room wall. Then she enlarged it by increasing both the width and the length by x feet. This equation can be solved to find x, the number of feet the art teacher increased each dimension of her picture.**

$$x^2 + 7x - 18 = 0$$

The area of the enlarged picture was 35 square feet. Which dimensions could be for the picture before it was enlarged?

A. 3 feet by 6 feet
B. 5 feet by 3 feet
C. 7 feet by 5 feet
D. 9 feet by 2 feet

[Key: B]

RATIONAL EQUATIONS

KEY IDEAS

1. An equation that contains one or more rational expressions is called a ***rational equation***.

2. Techniques for solving rational equations include steps that may give extraneous solutions that do not solve the original rational equation. Check each answer in the original problem to find and eliminate any extraneous solutions.

3. To solve a rational equation that is composed of one rational expression equal to another rational expression, find the common denominator and multiply both sides of the equation by it.

   Example:

   $\dfrac{4}{x+3} = \dfrac{7}{x}$ The common denominator is $x(x+3)$.

   $(x)(x+3)\left(\dfrac{4}{x+3}\right) = (x)(x+3)\left(\dfrac{7}{x}\right)$ Multiply both sides by the common denominator.

   $4(x) = 7(x+3)$ Simplify.

   $4x = 7x + 21$ Simplify.

   $4x - 7x = 7x + 21 - 7x$ Subtract $7x$ from each side.

   $-3x = 21$ Simplify.

   $\dfrac{-3x}{-3} = \dfrac{21}{-3}$ Divide each side by -3.

   $x = -7$

   Check the solution by substituting it into the original problem.

   $\dfrac{4}{x+3} = \dfrac{7}{x}$ Original problem.

   $\dfrac{4}{-7+3} = \dfrac{7}{-7}$ Substitute answer into problem.

   $\dfrac{4}{-4} = \dfrac{7}{-7}$ Simplify.

   $-1 = -1$ Simplify.

4.   To solve a rational equation that is composed of two or more rational expressions being combined on the same side of the equation, find the common denominator and build the expressions so the terms have common denominators. Then, combine the rational expressions.

Example:

What is the solution to the equation $\dfrac{3}{a} + \dfrac{1}{2a} = 9$?

Solution:

One way to do this is to find the common denominator and add the fractions to the left of the equal sign.

$\dfrac{3}{a} + \dfrac{1}{2a} = 9$ The common denominator for the fractions is $2a$.

$\dfrac{3}{a}\left(\dfrac{2}{2}\right) + \dfrac{1}{2a} = 9$ Multiply the first fraction by a fraction that equals one

to build a common denominator. (The second fraction already has the common denominator.)

$\dfrac{6}{2a} + \dfrac{1}{2a} = 9$ Multiply.

$\dfrac{7}{2a} = 9$ Combine the numerators but keep the denominator the same.

$(2a)\left(\dfrac{7}{2a}\right) = (2a)(9)$ Multiply both sides of the equation by $2a$.

$7 = 18a$

$\dfrac{7}{18} = \dfrac{18a}{18}$ Divide both sides of the equation by 18.

$a = \dfrac{7}{18}$

Check the solution by substituting it into the original problem.

$\dfrac{3}{\frac{7}{18}} + \dfrac{1}{2\left(\frac{7}{18}\right)} = 9$

$\dfrac{54}{7} + \dfrac{18}{14} = 9$

$\dfrac{54}{7} + \dfrac{9}{7} = 9$

$\dfrac{63}{7} = 9$

$9 = 9$

REVIEW EXAMPLES

1) Jenna had 10 red marbles and 20 blue marbles in a bag.

- She added x red marbles and x blue marbles to the bag.
- She took one-fourth of the red marbles and one-half of the blue marbles out of the bag.
- The total number of marbles she took out of the bag was 17.

This equation can be used to find x, the number of red marbles or the number of blue marbles that Jenna took out of the bag.

$$\frac{x+10}{4} + \frac{x+20}{2} = 17$$

What was the total number of red and blue marbles that were left in the bag?

Solution:

$\dfrac{x+10}{4} + \dfrac{x+20}{2} = 17$ The LCD is 4.

$4\left(\dfrac{x+10}{4}\right) + 4\left(\dfrac{x+20}{2}\right) = 4(17)$ Multiply through by the LCD.

$x + 10 + 2x + 40 = 68$ Simplify.

$3x + 50 = 68$ Simplify.

$3x + 50 - 50 = 68 - 50$ Subtract 50 from both sides of the equation.

$3x = 18$ Simplify.

$x = 6$

The value of x is 6. This means she had 10 + 6, or 16, blue marbles, and 20 + 6, or 26, red marbles before she removed any. That was a total of 16 + 26, or 42 marbles. Then she removed 17 marbles, so she had 42 − 17, or 25 red and blue marbles left in the bag.

2) What value of x makes this equation true?

$$\frac{4}{x-2} = \frac{3}{2x+1}$$

Solution:

$$\frac{4}{x-2} = \frac{3}{2x+1}$$

$$(x-2)(2x+1)\left(\frac{4}{x-2}\right) = (x-2)(2x+1)\left(\frac{3}{2x+1}\right) \qquad \text{Multiply both sides by the LCD.}$$

$4(2x+1) = 3(x-2)$ Simplify.

$8x+4 = 3x-6$ Simplify.

$8x+4-4 = 3x-6-4$ Subtract 4 from both sides.

$8x-3x = 3x-10-3x$ Subtract $3x$ from both sides.

$5x = -10$ Simplify.

$x = -2$

The solution is $x = -2$. Check to be sure that -2 does not result in 0 in the denominator. If the solution results in 0 when substituted into the original equation, -2 is not a valid solution.

EOCT Test Prep Items

1) **Breanne is rowing a boat at a rate of 5 miles per hour. She can row 7 miles downstream, with the current, in the same amount of time it takes her to row 3 miles upstream, against the current. This equation can be used to find the speed of the current in the stream.**

$$\frac{7}{5+c} = \frac{3}{5-c}$$

   What is the speed of the current in the stream?

   A. 2 miles per hour
   B. 3 miles per hour
   C. 4 miles per hour
   D. 5 miles per hour

[Key: A]

2) **What value of *x* makes this equation true?**

$$\frac{4}{x} + \frac{1}{2x} = 8$$

   A. $\dfrac{5}{24}$

   B. $\dfrac{9}{16}$

   C. $\dfrac{5}{8}$

   D. $\dfrac{4}{5}$

[Key: B]

RADICAL EQUATIONS

KEY IDEAS

1.   An equation that has a radical with a variable under the radicand is called a *radical equation*.

2.   If the equation $x = y$ is true, then the equation $x^2 = y^2$ is true.

3.   To solve a radical equation in the form $\sqrt{x} + b = c$, isolate the radical expression on one side of the equation. Then square both sides to eliminate the radical symbol.

   Example:

   Solve the equation $\sqrt{x} - 15 = 3$.

   Solution:

   $\sqrt{x} - 15 + 15 = 3 + 15$ Add 15 to both sides of the equation.

   $\sqrt{x} = 18$

   $\left(\sqrt{x}\right)^2 = (18)^2$ Square both sides of the equation.

   $x = 324$

   The solution is $x = 324$.

   Check your answer by substituting it back into the original equation.

   $\sqrt{x} - 15 = 3$

   $\sqrt{324} - 15 = 3$

   $18 - 15 = 3$

   $3 = 3$ This is a true equation, so the answer must be correct.

4.   When both sides of an equation are squared in the process of solving it, the result may be extraneous solutions that do not solve the original equation. Check each answer in the original problem to eliminate any extraneous solutions.

REVIEW EXAMPLES

1) Solve the equation $15 = 6 + \sqrt{x}$.

Solution:

$$15 = 6 + \sqrt{x}$$
$$15 - 6 = 6 + \sqrt{x} - 6 \qquad \text{Isolate the radical.}$$
$$9 = \sqrt{x}$$
$$(9)^2 = \left(\sqrt{x}\right)^2 \qquad \text{Square both sides.}$$
$$x = 81$$

Check:

$$15 = 6 + \sqrt{x}$$
$$15 = 6 + \sqrt{81}$$
$$15 = 6 + 9$$
$$15 = 15$$

Since 81 satisfies the original equation, then 81 is the solution.

2) Solve the equation $\sqrt{4x} - 9 = 25$.

Solution:

$$\sqrt{4x} - 9 = 25$$
$$\sqrt{4x} - 9 + 9 = 25 + 9 \qquad \text{Isolate the radical.}$$
$$\sqrt{4x} = 34$$
$$\left(\sqrt{4x}\right)^2 = (34)^2 \qquad \text{Square both sides.}$$
$$4x = 1156$$
$$\frac{4x}{4} = \frac{1156}{4} \qquad \text{Divide to isolate } x.$$
$$x = 289$$

Check:

$$\sqrt{4x} - 9 = 25$$

$$\sqrt{4(289)} - 9 = 25$$

$$\sqrt{1156} - 9 = 25$$

$$34 - 9 = 25$$

$$25 = 25$$

Since 289 satisfies the original equation, then 289 is the solution.

3) Solve the equation $\sqrt{b} + 5 = 2$.

Solution:

$$\sqrt{b} + 5 = 2$$

$$\sqrt{b} + 5 - 5 = 2 - 5 \qquad \text{Isolate the radical.}$$

$$\sqrt{b} = -3$$

$$\left(\sqrt{b}\right)^2 = (-3)^2 \qquad \text{Square both sides.}$$

$$b = 9$$

Check:

$$\sqrt{b} + 5 = 2$$

$$\sqrt{9} + 5 \neq 2$$

$$3 + 5 \neq 2$$

$$8 \neq 2$$

Since 9 does not satisfy the original equation, then 9 is not a solution. This equation has no real solution.

EOCT Test Prep Items

1) **The base of a triangle is represented by** $\sqrt{x}$ **inches. The height is 4 inches. The area of the triangle is 18 square inches.**

   What is the value of x**?**

   A. 9
   B. 18
   C. 36
   D. 81

   [Key: D]

2) **The equation** $\sqrt{3x+4}=5$ **compares the areas of two congruent triangles. What is the value of** x**?**

   A. 1
   B. 3
   C. 7
   D. 25

   [Key: C]

CHARACTERISTICS OF FUNCTIONS AND THEIR GRAPHS

KEY IDEAS

1. We call *f* an ***even function*** if $f(x) = f(-x)$ for all values in its domain.

   Example:

   Suppose *f* is an even function and the point (2, 7) is on the graph of *f*. Name one other point that must be on the graph of *f*.
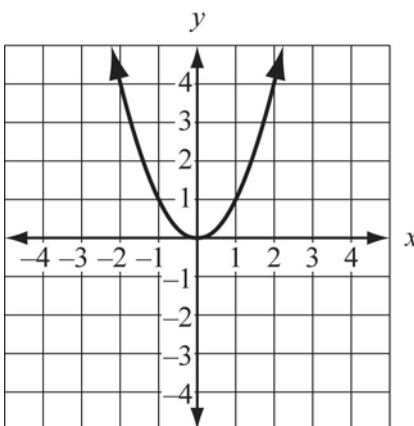
   Since (2, 7) is on the graph, 2 is in the domain and $f(2) = 7$. By definition of an even function, $f(-2) = f(2) = 7$.

   Therefore, (−2, 7) is also on the graph of *f*.

2. The graph of an even function has line symmetry with respect to the *y*-axis.

   Example:

   This is the graph of the function $y = x^2$, which is an even function.

   Notice that for any number *b*, the points (*x*, *b*) and (−*x*, *b*) are at the same height on the grid and are equidistant from the *y*-axis. That means they represent line symmetry with respect to the *y*-axis.

3. We call f an ***odd function*** if $f(-x) = -f(x)$ for all values in its domain.

   Example:

   Suppose f is an odd function and the point $(-2, 8)$ is on the graph of f. Name one other point that must be on the graph of f.

   Since $(-2, 8)$ is on the graph, -2 is in the domain and $f(-2) = 8$. By definition of an odd function, $-(-2)$, or 2, is also in the domain and $f(2) = -f(-2) = -8$.

   Therefore, $(2, -8)$ is also on the graph of f.

4. This is the graph of the function $y = x^3$, which is an odd function.
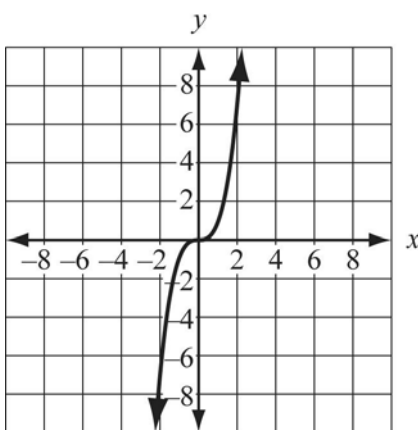
   The graph of an odd function has rotational symmetry of 180° about the origin. This is also called symmetry with respect to the origin. Whenever the graph of an odd function contains the point (a, b) it also contains the point $(-a, -b)$.

5. For any graph, ***rotational symmetry*** of 180° about the origin is the same as point symmetry of reflection through the origin. Reflecting a point through the x-axis and then through the y-axis gives the same final point as rotating it 180° about the origin.

6. One way to solve a quadratic equation is to graph the corresponding quadratic function on a coordinate plane. The standard form of a quadratic equation is $ax^2 + bx + c = 0$. The corresponding function is $f(x) = ax^2 + bx + c$. The ***solutions***, or ***roots***, of the equation $ax^2 + bx + c = 0$ can be determined by finding the values of x where the graph of the function crosses the x-axis, otherwise known as the x-intercepts. Note that in Mathematics I, the only quadratics being addressed are those where a = 1.

A quadratic equation may have one solution, two solutions, or no solution.

- If the graph crosses the *x*-axis at only one point (has only one *x*-intercept), then the equation has only one unique solution. This is also known as a double root because the factors of the quadratic are identical and produce the same value.
- If the graph crosses the *x*-axis at two points (has two *x*-intercepts), then the equation has two solutions.
- If the graph does **not** cross the *x*-axis at any point (has no *x*-intercept), then the equation has no solution.

One Solution   **Two Solutions**   **No Solution**
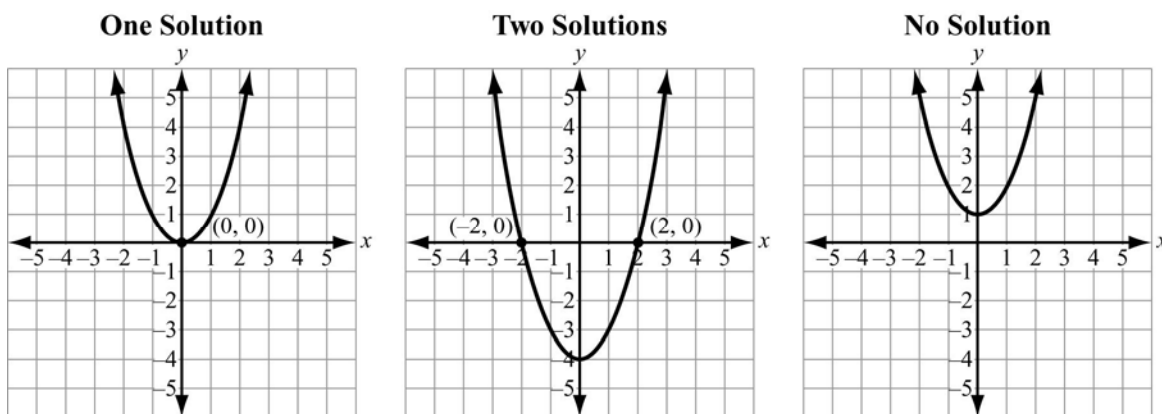
7. The graph of an equation can be reflected through the *x*-axis or the *y*-axis. If an equation is in the form $y = f(x)$, then:
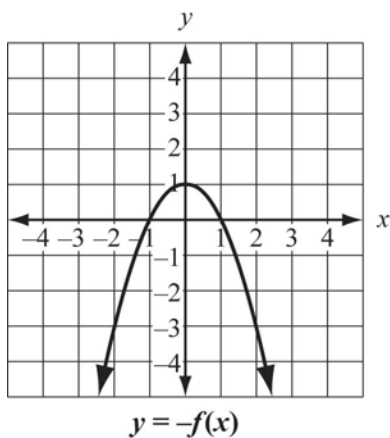
- to reflect the graph of $y = f(x)$ through the *x*-axis, form the equation $y = -f(x)$.
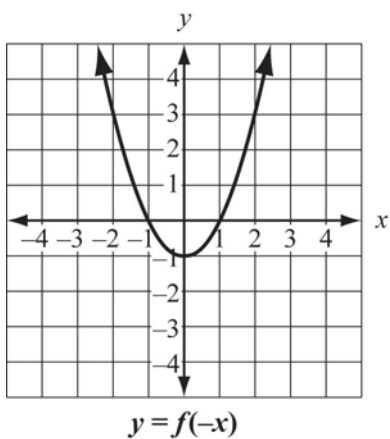- to reflect the graph of $y = f(x)$ through the *y*-axis, form the equation $y = f(-x)$.

Example:

Consider the equation $y = x^2 - 1$. Let $f(x) = x^2 - 1$ and then $y = f(x)$. This is the graph of $y = f(x)$.

$y = f(x)$
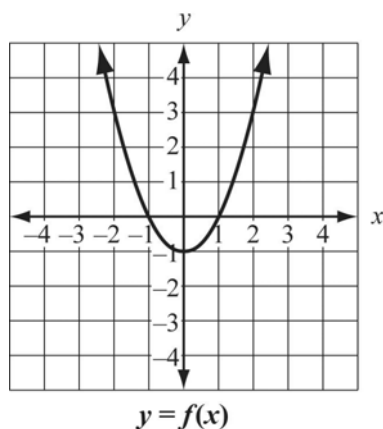
This is the graph of the reflection of the equation $y = f(x)$ through the x-axis.

$y = -f(x)$

This is the graph of the reflection of the equation $y = f(x)$ through the y-axis.

$y = f(-x)$

Notice that this reflection looks the same as the original graph because the original graph was symmetric with respect to the y-axis.

REVIEW EXAMPLES

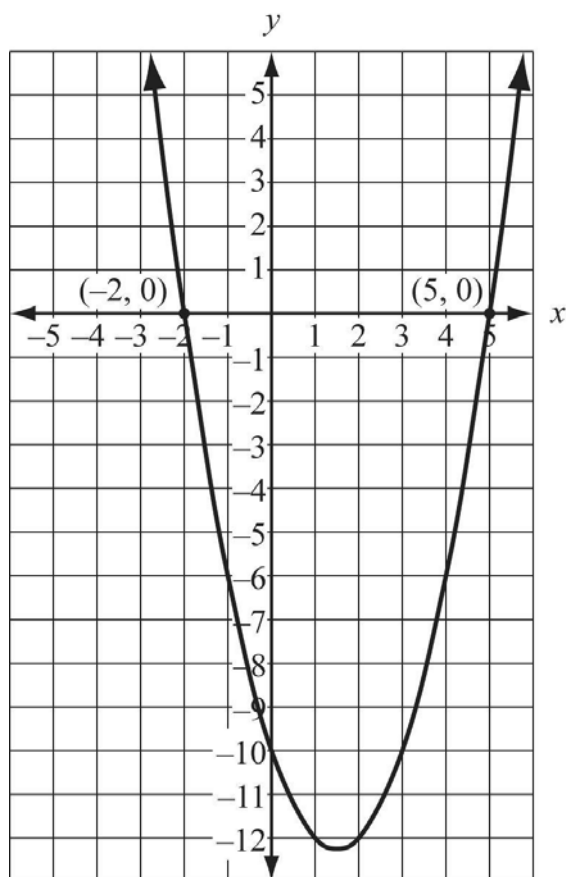1) Solve the equation $x^2 - 3x - 10 = 0$ by graphing.

Solution:

Rewrite the equation in function form.

$$y = x^2 - 3x - 10$$

Set up an x/y table for the function.

x	−3	−2	−1	0	1	2	3	4	5
y	8	0	−6	−10	−12	−12	−10	−6	0

Graph the function from the table.

The points where the graph crosses the x-axis are (−2, 0) and (5, 0). The numbers −2 and 5 are solutions to the quadratic equation $x^2 - 3x - 10 = 0$ because when each is substituted for x, the result is 0.

2) Solve the equation $x^2 + 7x + 12 = 0$.

Solution:

Rewrite the equation in function form.

$$y = x^2 + 7x + 12$$

Set up an *x/y* table for the function.

x	−5	−4	−3	−2	−1	0
y	2	0	0	2	6	12

A quadratic equation can have zero, one, or two solutions. A solution is a point where the graph of the equation crosses the *x*-axis. In the table, that is a point that has a *y*-value of 0. In this example, the points where the graph crosses the *x*-axis are (−4, 0) and (−3, 0). The numbers −4 and −3 are solutions to the quadratic equation $x^2 + 7x + 12 = 0$.

Be sure to select enough values for *x* so that you can see the pattern in the table where the parabola formed by the quadratic equation changes direction. Select enough points on either side of the turning point to see whether the graph crosses the *x*-axis zero, one, or two times.

EOCT Test Prep Items

1) **The graph of the function $f(x) = x^3 + x^2 + 4$ is shown on this coordinate plane.**
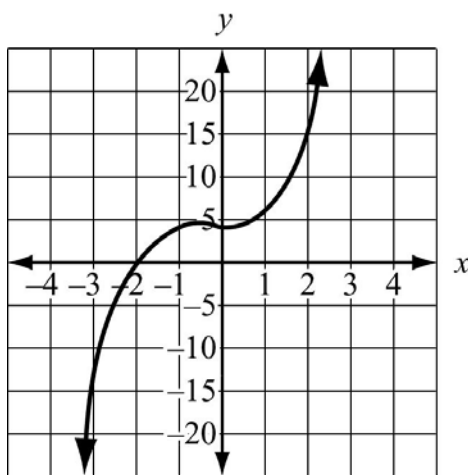
Which statement best describes the behavior of the function within the interval $x = -3$ to $x = 0$?

A. From left to right, the function rises only.
B. From left to right, the function falls and then rises.
C. From left to right, the function rises and then falls.
D. From left to right, the function falls, rises, and then falls.

[Key: C]

2) **What is the formula for the function with a graph the same as the graph obtained by reflecting the graph of $y = \sqrt{-x}$ across the x-axis?**

A. $y = -\sqrt{-x}$
B. $y = -\sqrt{x}$
C. $y = \sqrt{-x}$
D. $y = \sqrt{x}$

[Key: A]

Unit 6: Coordinate Geometry

This unit investigates the properties of geometric figures on the coordinate plane. Students develop and use the formulas for the distance between two points, the distance between a point and a line, and the midpoint of segments. Many topics that were addressed in previous units will be revisited relative to the coordinate plane.

KEY STANDARDS

MM1G1. Students will investigate properties of geometric figures in the coordinate plane.

    a. Determine the distance between two points.
    b. Determine the distance between a point and a line.
    c. Determine the midpoint of a segment.
    d. Understand the distance formula as an application of the Pythagorean theorem.
    e. Use the coordinate plane to investigate properties of and verify conjectures related to triangles and quadrilaterals.

DISTANCES IN THE COORDINATE PLANE

KEY IDEAS

1. On a coordinate plane, the distance between two points that lie on the same vertical line or two points that lie on the same horizontal line can be found by either subtracting the *y*-coordinates of the two points on the vertical line or the *x*-coordinates of the two points on the horizontal line and taking the absolute value.

    Example:

    Consider $\triangle ABC$ shown on this coordinate plane.
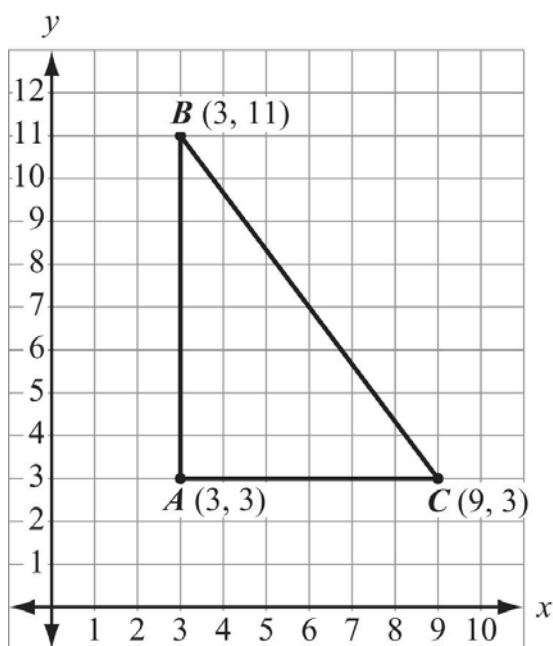
    To find the distance between points *A* and *B*, subtract the *y*-coordinates of the two points and take the absolute value.

    The distance between *A* and *B*, or the length of $\overline{AB}, = |y_2 - y_1| = |11 - 3| = |8| = 8$, or 8 units.

    To find the distance between points *A* and *C*, subtract the *x*-coordinates of the two points and take the absolute value.

    The distance between *A* and *C*, or the length of $\overline{AC}, = |x_2 - x_1| = |3 - 9| = |-6| = 6$, or 6 units.

2.   One way to find the distance between two points that are not on the same horizontal or
      vertical line on a coordinate plane is to use the **Pythagorean theorem**.

      Example:

      To find the distance between points B and C in $\triangle ABC$ from Key Idea **#1**, first notice that
      $\triangle ABC$ is a right triangle with legs $\overline{AB}$ and $\overline{AC}$ and hypotenuse $\overline{BC}$. The distance between
      points B and C is also the length of the hypotenuse.

      The length of $\overline{AB}$ is 8 units. The length of $\overline{AC}$ is 6 units. Let h represent the length of the
      hypotenuse. Then use the Pythagorean theorem to calculate its length.

$$h^2 = 8^2 + 6^2$$
$$h^2 = 64 + 36$$
$$h^2 = 100$$
$$h = \sqrt{100}$$
$$h = 10 \text{ units}$$

      The distance between B and C, or the length of $\overline{BC}$, is 10 units.

3.   Another way to find the distance between two points that are not on the same horizontal or
      vertical line on a coordinate plane is to use the **distance formula**, which is an application
      of the Pythagorean theorem on the coordinate plane. The distance formula states that on a
      coordinate plane the distance, d, between any two points (x_1, y_1) and (x_2, y_2) is defined as
      $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

      Example:

      To find the distance between B and C, or the length of $\overline{BC}$ from the example in Key Idea
      #1, use the distance formula and find
      $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 9)^2 + (11 - 3)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$, or 10 units.

      The distance formula allows the distance between any two points to be found by using
   only the coordinates of those two points.

4.   A **perpendicular** line segment from a point to a line is the shortest segment from the point
      to the line.

      Recall these facts.
      - Two lines are perpendicular if they intersect to form four right, or 90°, angles.
      - The product of the slopes of two perpendicular lines is –1.

Example:

The lines represented by $y = 2x + 1$ and $y = -\frac{1}{2}x - 1$ are perpendicular.
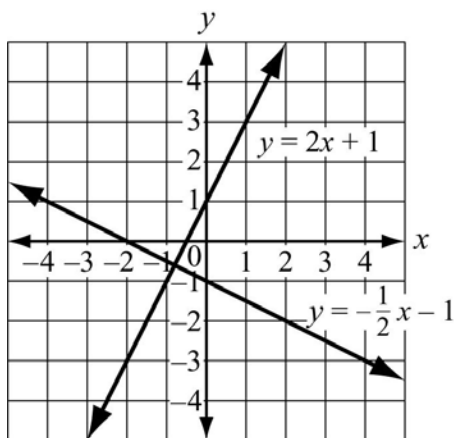
The two lines intersect to form four right angles.

The product of the slopes of the two lines is $(2)\left(-\frac{1}{2}\right)$, or -1.

5. The midpoint of a line segment is the point that divides the segment into two congruent

    parts. The **_midpoint formula_** states that the midpoint of a segment with endpoints (x_1, y_1)

    and (x_2, y_2) is the point that has the coordinates $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Example:

The coordinates of the endpoints of $\overline{ST}$ are $S(-8, 4)$ and $T(-2, -4)$. To find the coordinates
of the midpoint of $\overline{ST}$, substitute the coordinates of the endpoints into the midpoint
formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$\left(\frac{-8 + -2}{2}, \frac{4 + -4}{2}\right)$$
$$\left(\frac{-10}{2}, \frac{0}{2}\right)$$
$$(-5, 0)$$

The midpoint of $\overline{ST}$ is the point with the coordinates $(-5, 0)$.

REVIEW EXAMPLES

1) Quadrilateral *QRST* has vertices at $Q(4, 4)$, $R(6, 6)$, $S(4,-8)$, and $T(-4,6)$. What is the length of side $\overline{RS}$?

Solution:

Use the two points $R(6, 6)$ and $S(4,-8)$ as (x_1, y_1) and (x_2, y_2), respectively. Substitute the coordinates into the distance formula.

$$\overline{RS} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\overline{RS} = \sqrt{(4-6)^2 + (-8-6)^2}$$
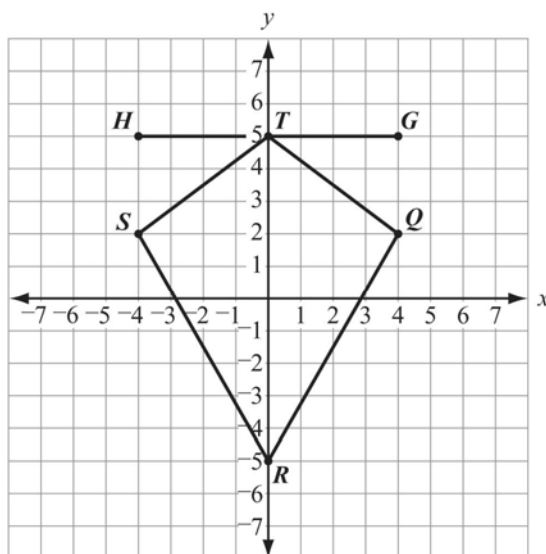$$\overline{RS} = \sqrt{(-2)^2 + (-14)^2}$$
$$\overline{RS} = \sqrt{4+196}$$
$$\overline{RS} = \sqrt{200}$$
$$\overline{RS} = 10\sqrt{2}$$

The length of segment RS is $10\sqrt{2}$ units, or approximately 14.1 units.

2) The coordinate grid below shows a sketch of a silver pendant.

Each unit on the grid represents 10 millimeters.
   a. What is the shortest distance between point R and $\overline{HG}$?
   b. How many millimeters longer is $\overline{RT}$ than $\overline{ST}$?

Solution:

a. The shortest distance between point R and $\overline{HG}$ is the length of $\overline{RT}$ because $\overline{RT} \perp \overline{HG}$. Point R is located at $(0, -5)$. Point T is located at $(0, 5)$. To find the distance, subtract the y-coordinates of these two points and take the absolute value of the difference.

The length of $\overline{RT} = |y_2 - y_1| = |5 - (-5)| = |-10| = 10,$ or 10 units.

Since 1 unit = 10 mm, then 10 units = $10 \times 10 = 100$ mm.
The shortest distance is 100 mm.

b. From part (a), the length of $\overline{RT} = 100$ mm. To find the distance from S to T, use the Pythagorean theorem since the points STH form a right triangle $(\overline{SH} \perp \overline{HT})$.

$$\overline{ST}^2 = \overline{SH}^2 + \overline{TH}^2$$
$$\overline{ST} = \sqrt{\overline{SH}^2 + \overline{TH}^2}$$
$$\overline{ST} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Since 1 unit = 10 mm, then 5 units = $10 \times 5 = 50$ mm.
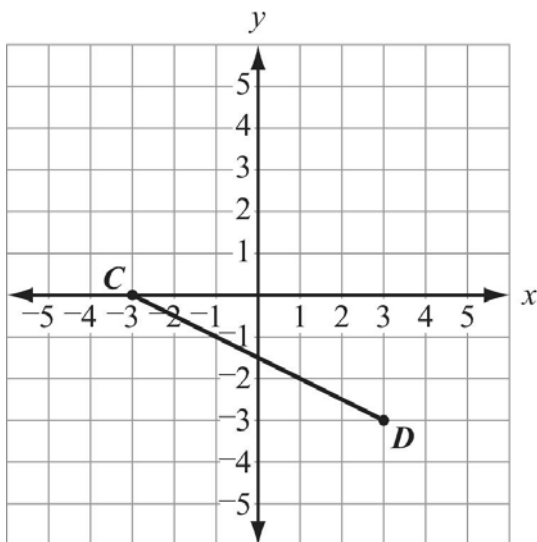So, $\overline{RT} - \overline{ST} = 100 - 50 = 50$ mm.

Note: The length of $\overline{ST}$ can also be found by using the distance formula. The length of $\overline{ST} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Point S is at $(-4, 2)$ and point T is at $(0, 5)$.

The length of $\overline{ST} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - (-4))^2 + (5 - 2)^2} = \sqrt{16 + 9} = 5$ units, or 50 mm.

3) This coordinate grid shows line segment *CD*.

Point *C* is the midpoint of line segment *BD*. What are the coordinates of point *B*?

Solution:

Point *C*, the midpoint, is located at $(-3, 0)$. Point *D*, one endpoint of line segment *BD*, is located at $(3, -3)$.
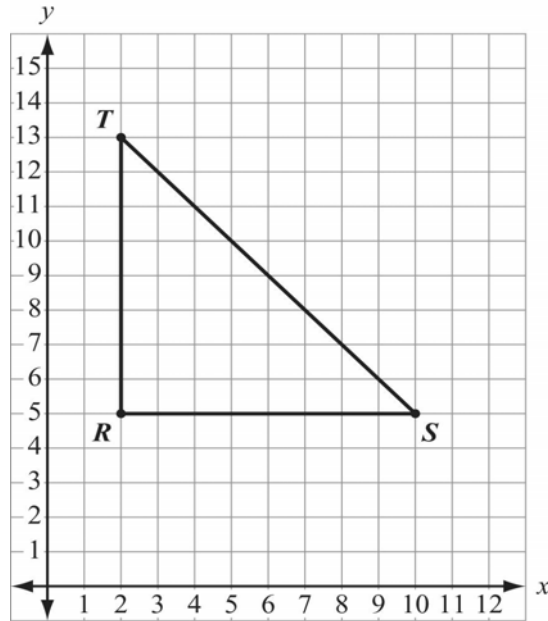
Substitute what is known into the *x*- and *y*-coordinates of the midpoint formula to find the coordinates of endpoint *B*.

$$\frac{x_1 + x_2}{2} = -3 \qquad\qquad \frac{y_1 + y_2}{2} = 0$$
$$\frac{3 + x_2}{2} = -3 \qquad\qquad \frac{-3 + y_2}{2} = 0$$
$$3 + x_2 = -6 \qquad\qquad -3 + y_2 = 0$$
$$x_2 = -9 \qquad\qquad\qquad y_2 = 3$$

Endpoint *B* is located at $(-9, 3)$.

EOCT Test Prep Items

1) **On this coordinate grid, the library is located at point *T*, the music store is located at point *S*, and the pet store is located at point *R*.**
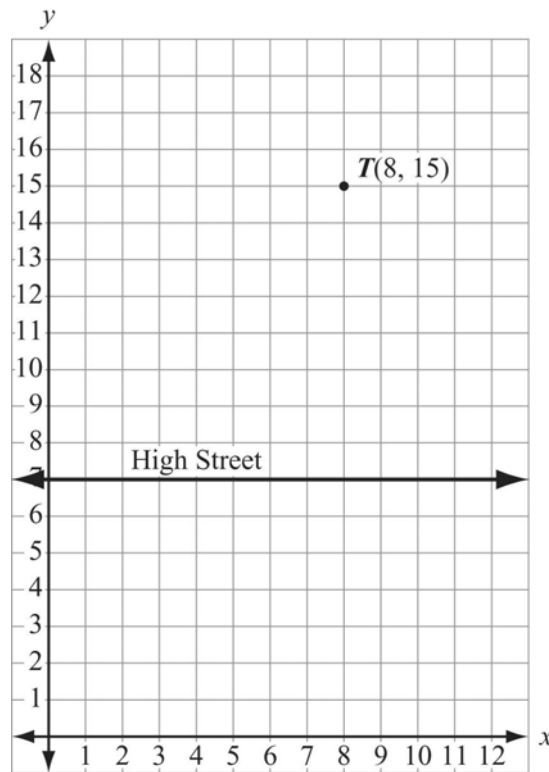

    Each grid line represents 1 mile. How much farther, to the nearest tenth of a mile, is the music store from the library than it is from the pet store?

    A. 3.3 miles
    B. 8.0 miles
    C. 11.3 miles
    D. 13.6 miles

[Key: A]

2) Wesley is walking with his dog along High Street. He wants to go from High Street to his house located at point *T* on this grid.

Once Wesley leaves High Street, he will have to walk across a rocky field. His dog refuses to walk across the field, so Wesley will have to carry him.

At what point on High Street should Wesley turn and walk toward his house so that he carries his dog the shortest possible distance?

A. (0, 7)
B. (7, 7)
C. (8, 7)
D. (10, 7)

[Key: C]

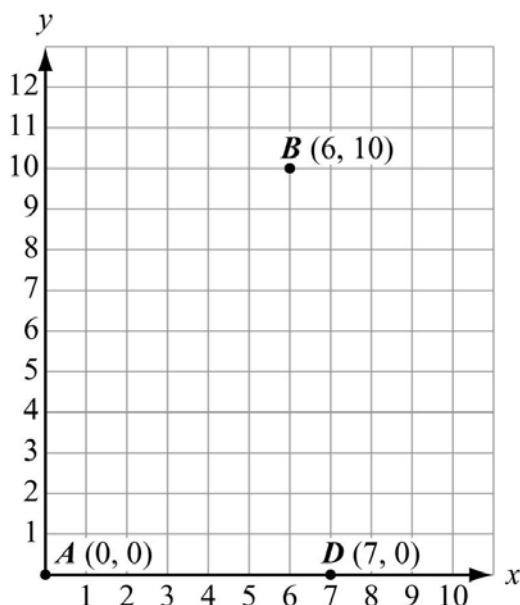# TRIANGLES AND QUADRILATERALS IN THE COORDINATE PLANE

KEY IDEAS

1. The coordinate plane can be used to determine or prove information regarding various classifications of triangles and quadrilaterals. Coordinate pairs in the coordinate plane identify the locations of points. These locations can be used in the distance and midpoint formulas. Additionally, the points can be used to find the slope. Slope can be used to determine parallel or perpendicular lines.

2. There are many different properties associated with triangles and quadrilaterals. Refer to **Unit 3** to review the properties, postulates, and theorems introduced there. The following is a summary of some important properties for your reference.

    -   Two lines are parallel if their slopes are equal.
    -   Two lines are perpendicular if the product of their slopes is −1.
    -   Triangles can be classified by their sides (scalene, equilateral, or isosceles) or by their angles (acute, right, or obtuse).
    -   Triangles can be proven congruent by using SSS, SAS, ASA, AAS, and HL theorems.
    -   The segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is half the length of the third side.
    -   Some quadrilaterals can be classified as squares, rectangles, trapezoids, kites, parallelograms, or rhombuses.
    -   If a quadrilateral is a parallelogram, then its opposite sides and opposite angles are congruent, its opposite sides are parallel, its consecutive angles are supplementary, and its diagonals bisect each other.
    -   A parallelogram is a rhombus if its diagonals are perpendicular, each of its diagonals bisects a pair of opposite angles, and all four of its sides are congruent.
    -   A parallelogram is a rectangle if its diagonals are congruent.
    -   A parallelogram is a square if its diagonals are perpendicular and congruent.
    -   A quadrilateral is an isosceles trapezoid if its base angles are congruent and its diagonals are congruent.
    -   The centroid is the point of concurrency of the medians of a triangle.
    -   The circumcenter is the point of concurrency of the perpendicular bisectors of the sides of a triangle.
    -   The incenter is the point of concurrency of the bisectors of the angles of a triangle.
    -   The orthocenter is the point of concurrency of the altitudes of a triangle.

REVIEW EXAMPLES

1) Three vertices of parallelogram *ABCD* are *A*(0, 0), *B*(6, 10), and *D*(7, 0). What are the coordinates of vertex *C*?

Solution:

Plot what you are given on a coordinate grid.

To determine the coordinates of vertex *C*, think about these facts regarding parallelograms.

- If a quadrilateral is a parallelogram, then its opposite sides and opposite angles are congruent.
- If a quadrilateral is a parallelogram, then its opposite sides are parallel.

This means that $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. The length of line segment $\overline{AD}$ is 7 units. Since $\overline{AD} \cong \overline{BC}$, then the length of line segment $\overline{BC} = 7$ units. Since $\overline{AD}$ lies on the horizontal axis, then $\overline{BC}$ must also be a horizontal line. The *x*-coordinate of vertex *C* is located 7 horizontal units from the *x*-coordinate of vertex *B*: 6 + 7 = 13.

The *x*-coordinate of vertex *C* is 13.

If *ABCD* is a parallelogram, then $\overline{AD} \parallel \overline{BC}$. Vertex *B* is 10 units from the *x*-axis, so vertex *C* must also be 10 units from the *x*-axis. The *y*-coordinate of vertex *C* is 10.

The coordinates of vertex *C* are (13, 10).

2) Line segment *CD*, shown on this coordinate grid, is the hypotenuse of right isosceles triangle *BCD*.
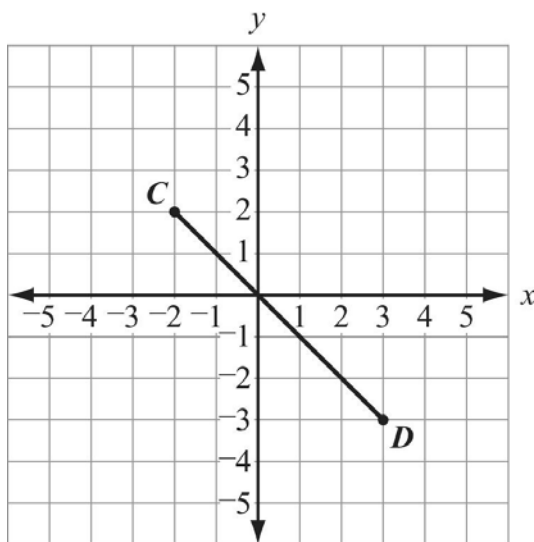
Identify two possible locations for vertex *B*.

Solution:

Since $\overline{CD}$ is the hypotenuse of $\triangle BCD$, then $\angle B$, the angle opposite the hypotenuse, is a right angle. This means that $\overline{CB} \perp \overline{BD}$. Since the triangle is an isosceles triangle, then $\overline{CB} \cong \overline{BD}$.

Extend a vertical line from point *C* and a horizontal line from point *D*. The lines intersect perpendicularly at point (–2, –3). The length of line segment *CB* is 5 units, and the length of line segment *BD* is 5 units. Since these two lines are the same length, this verifies that the triangle is isosceles.

Extend a horizontal line from point *C* and a vertical line from point *D*. The lines intersect perpendicularly at point (3, 2). The length of line segment *CB* is 5 units, and the length of line segment *BD* is 5 units. This verifies that the triangle is isosceles.

Point *B* could be located at either (–2, –3) or (3, 2).

EOCT Test Prep Items

1) **The vertices of quadrilateral *EFGH* have the coordinates $E(2,-2)$, $F(4,3)$, $G(-1,5)$, and $H(-3,0)$. Which of the following describes quadrilateral *EFGH*?**

   A. a square
   B. a rectangle that is not a square
   C. a rhombus that is not a square
   D. a parallelogram that is not a rectangle

   [Key: A]

2) **In isosceles $\triangle PQR$, $\overline{PQ} \cong \overline{QR}$. Point *P* is located at (1, 1). The centroid of the triangle is located at (6, 2).**

   Which coordinate pair could represent the location of point *R* of the triangle?

   A. (1, 4)
   B. (5, 1)
   C. (7, 3)
   D. (11, 1)

   [Key: D]

Appendix A
EOCT Sample Overall Study Plan Sheet

Here is a sample of what an OVERALL study plan might look like. You can use the Blank Overall Study Plan Sheet in Appendix B or create your own.

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. *This study guide*
2. *Pens*
3. *Highlighter*
4. *Notebook*
5. *Dictionary*
6. *Calculator*
7. *Mathematics textbook*

Possible Study Locations:

- First choice: *The library*

- Second choice: *My room*

- Third choice: *My mom's office*

Overall Study Goals:

1. *Read and work through the entire study guide.*
2. *Answer the sample questions and study the answers.*
3. *Do additional reading in a mathematics textbook.*

Number of Weeks I Will Study: *6 weeks*

Number of Days a Week I Will Study: *5 days a week*

Best Study Times for Me:

- Weekdays: *7:00 p.m. – 9:00 p.m.*
- Saturday: *9:00 a.m. – 11:00 a.m.*
- Sunday: *2:00 p.m. – 4:00 p.m.*

Appendix B
Blank Overall Study Plan Sheet

Materials/Resources I May Need When I Study:
(You can look back at page 2 for ideas.)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

Possible Study Locations:

- First choice: _____

- Second choice _____

- Third choice _____

Overall Study Goals:

1. _____

2. _____

3. _____

4. _____

5. _____

Number of Weeks I Will Study: _____

Number of Days a Week I Will Study: _____

Best Study Times for Me:

- Weekdays: _____

- Saturday: _____

- Sunday: _____

Appendix C
EOCT Sample Daily Study Plan Sheet

Here is a sample of what a DAILY study plan might look like. You can use the Blank Daily Study Plan Sheet in Appendix D or create your own.

Materials I May Need Today:

1. *Study guide*
2. *Pen/pencil*
3. *Notebook*

Today's Study Location: *The desk in my room*

Study Time Today: *From 7:00 p.m. to 8:00 p.m. with a short break at 7:30 p.m.*
(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes?
30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything beyond 40 minutes may not be productive—you will most likely fidget and daydream your time away. "Doing time" at your desk doesn't count as real studying.)

If I Start to Get Tired or Lose Focus Today, I Will: *Do some sit-ups*

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages, units, or standards. The more specific you are, the better able you will be to tell if you reached your goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a time.)

Study Task	Completed	Needs More Work	Needs More Information
1. Review what I learned last time	X		
2. Study the first main topic in Unit 1	X		
3. Study the second main topic in Unit 1		X	

What I Learned Today:

1. *Reviewed basic functions*
2. *The importance of checking that the answer "makes sense" by estimating first*
3. *How to use math symbols*

Today's Reward for Meeting My Study Goals: *Eating some popcorn*

Appendix D
Blank Daily Study Plan Sheet

Materials I May Need Today:

    1. _____

    2. _____

    3. _____

    4. _____

    5. _____

Today's Study Location: _____

Study Time Today: _____
(Be sure to consider how long you can actively study in one sitting. Can you sit for 20 minutes?
30 minutes? An hour? If you say you will study for three hours, but get restless after 40 minutes, anything
beyond 40 minutes may not be productive—you will most likely fidget and daydream your time away.
"Doing time" at your desk doesn't count as real studying.)

If I Start To Get Tired or Lose Focus Today, I Will: _____

Today's Study Goals and Accomplishments: (Be specific. Include things like number of pages,
sections, or standards. The more specific you are, the better able you will be to tell if you reached your
goals. Keep it REALISTIC. You will retain more if you study small "chunks" or blocks of material at a
time.)

Study Task	Completed	Needs More Work	Needs More Information
1.			
2.			
3.			
4.			
5.			

What I Learned Today:

    1. _____

    2. _____

    3. _____

Today's Reward for Meeting My Study Goals: _____