Arizona's Common Core Standards
Mathematics

Standards - Mathematical Practices - Explanations and Examples

High School Grades 9th – 12th

ARIZONA DEPARTMENT OF EDUCATION
HIGH ACADEMIC STANDARDS FOR STUDENTS
State Board Approved June 2010
January 2013 Publication

High School (9th – 12th) Overview

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in fourth courses or advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students. There are two pathways that exist for course development, the traditional pathway and the integrated pathway. Standards labeled A I (Algebra I), G (Geometry), and A II (Algebra II) are included in courses in the traditional pathway. Standards labeled M I, M II, and M III are included in courses in the integrated pathway.

The high school standards are listed in conceptual categories including Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability, and Contemporary Mathematics.

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

Number and Quantity
- The Real Number System (N-RN)
- Quantities (N-Q)
- The Complex Number System (N-CN)
- Vector and Matrix Quantities (N-VM)

Algebra
- Seeing Structure in Expressions (A-SSE)
- Arithmetic with Polynomials and Rational Expressions (A-APR)
- Creating Equations (A-CED)
- Reasoning with Equations and Inequalities (A-REI)

Functions
- Interpreting Functions (F-IF)
- Building Functions (F-BF)
- Linear, Quadratic, and Exponential Models (F-LE)
- Trigonometric Functions (F-TF)

Geometry
- Congruence (G-CO)
- Similarity, Right Triangles, and Trigonometry (G-SRT)
- Circles (G-C)
- Expressing Geometric Properties with Equations (G-GPE)
- Geometric Measurement and Dimension (G-GMD)
- Modeling with Geometry (G-MG)

Modeling

Statistics and Probability
- Interpreting Categorical and Quantitative Data (S-ID)
- Making Inferences and Justifying Conclusions (S-IC)
- Conditional Probability and the Rules of Probability (S-CP)
- Using Probability to Make Decisions (S-MD)

Contemporary Mathematics
- Discrete Mathematics (CM-DM)

High School: Number and Quantity Overview

The Real Number System (N-RN)

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities (N-Q)

- Reason quantitatively and use units to solve problems

The Complex Number System (N-CN)

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities (N-VM)

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Number and Quantity

Numbers and the Number System

During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3.... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity: The Real Number System (N-RN)

Extend the properties of exponents to rational exponents

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-RN.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.* Connections: *11-12.RST.4; 11-12.RST.9; 11-12.WHST.2d*	A II	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others.	Students may explain orally or in written format.
HS.N-RN.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	A II	M II	*HS.MP.7.* Look for and make use of structure.	**Examples:** • $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$; $5^{\frac{2}{3}} = \sqrt[3]{5^2}$ • Rewrite using fractional exponents: $\sqrt[5]{16} = \sqrt[5]{2^4} = 2^{\frac{4}{5}}$ • Rewrite $\frac{\sqrt{x}}{x^2}$ in at least three alternate forms. Solution: $x^{-\frac{3}{2}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{\sqrt{x^3}} = \frac{1}{x\sqrt{x}}$ • Rewrite $\sqrt[4]{2^{-4}}$ .using only rational exponents. • Rewrite $\sqrt[3]{x^3 + 3x^2 + 3x + 1}$ in simplest form.

Number and Quantity: The Real Number System (N-RN)

Use properties of rational ad irrational numbers

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-RN.3. Explain why the sum or product of two rational numbers are rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Connection: *9-10.WHST.1e*	A I	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others.	Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition). **Example:** • Explain why the number 2π must be irrational, given that π is irrational. Answer: if 2π were rational, then half of 2π would also be rational, so π would have to be rational as well.

Number and Quantity: Quantities ★ (N-Q)

Reason qualitatively and units to solve problems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Connections: *SCHS-S1C4-02*; *SSHS-S5C5-01*	A I ★	M I ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour: $24000\,\text{sec} \bullet \frac{1\text{min}}{60\text{sec}} \bullet \frac{1\text{hr}}{60\text{min}} \bullet \frac{1\text{day}}{24\text{hr}}$ which is more than 8 miles per hour. Graphical representations and data displays include, but are not limited to: line graphs, circle graphs, histograms, multi-line graphs, scatterplots, and multi-bar graphs.
HS.N-Q.2. Define appropriate quantities for the purpose of descriptive modeling. Connection: *SSHS-S5C5-01*	A I A II ★	M I M II M III ★	*HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision.	**Examples:** • What type of measurements would one use to determine their income and expenses for one month? • How could one express the number of accidents in Arizona?
HS.N-Q.3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	A I	M I ★	*HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	The margin of error and tolerance limit varies according to the measure, tool used, and context. **Example:** • Determining price of gas by estimating to the nearest cent is appropriate because you will not pay in fractions of a cent but the cost of gas is $\frac{\$3.479}{gallon}$.

Number and Quantity: The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-CN.1. Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	A II	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.6.* Attend to precision.	
HS.N-CN.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. Connection: *11-12.RST.4*	A II	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	**Example:** • Simplify the following expression. Justify each step using the commutative, associative and distributive properties. $(3-2i)(-7+4i)$ Solutions may vary; one solution follows: $(3-2i)(-7+4i)$ $3(-7+4i)-2i(-7+4i)$ Distributive Property $-21+12i+14i-8i^2$ Distributive Property $-21+(12i+14i)-8i^2$ Associative Property $-21+i(12+14)-8i^2$ Distributive Property $-21+26i-8i^2$ Computation $-21+26i-8(-1)$ $i^2=-1$ $-21+26i+8$ Computation $-21+8+26i$ Commutative Property $-13+26i$ Computation

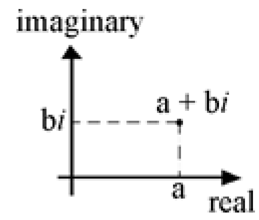
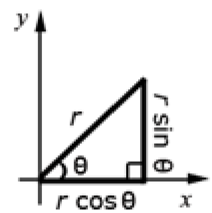
Number and Quantity: The Complex Number System (N-CN)

Perform arithmetic operations with complex numbers *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-CN.3. Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. Connection: *11-12.RST.3*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	**Example:** • Given $w = 2 - 5i$ and $z = 3 + 4i$ *a.* Use the conjugate to find the modulus of *w*. *b.* Find the quotient of *z* and *w*. Solution: a. $\lvert w \rvert^2 = w\,\overline{w}$ $\lvert w \rvert^2 = (2-5i)(2+5i)$ $\lvert w \rvert^2 = 4 + 10i - 10i - 25i^2$ $\lvert w \rvert^2 = 4 - 25i^2$ $\lvert w \rvert^2 = 4 - 25(-1)$ $\lvert w \rvert^2 = 4 + 25$ $\lvert w \rvert^2 = 29$ $\lvert w \rvert = \sqrt{29}$ b. $\frac{z}{w} = \frac{3+4i}{2-5i}$ $\frac{z}{w} = \frac{3+4i}{2-5i}\left(\frac{2+5i}{2+5i}\right)$ $\frac{z}{w} = \frac{6+15i+8i-20}{4+25}$ $\frac{z}{w} = \frac{-14+23i}{29}$

Number and Quantity: The Complex Number System (N-CN)

Represent complex numbers and their operations on the complex plane

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-CN.4. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. Connection: *11-12.RST.3*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	Students will represent complex numbers using rectangular and polar coordinates. $a + bi = r(\cos \vartheta + \sin \vartheta)$  **Examples:** • Plot the points corresponding to $3 - 2i$ and $1 + 4i$. Add these complex numbers and plot the result. How is this point related to the two others? • Write the complex number with modulus (absolute value) 2 and argument $\pi/3$ in rectangular form. • Find the modulus and argument ($0 < \theta < 2\pi$) of the number $\sqrt{6} + \sqrt{-6}$.
HS.N-CN.5. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* *$(-1 + \sqrt{3}\, i)^3 = 8$ because* *$(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	

Number and Quantity: The Complex Number System (N-CN)

Represent complex numbers and their operations on the complex plane *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-CN.6. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. Connection: *11-12.RST.3*	+	+	*HS.MP.2.* Reason abstractly and quantitatively.	

Number and Quantity: The Complex Number System (N-CN)

Use complex numbers in polynomial identities and equations

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-CN.7. Solve quadratic equations with real coefficients that have complex solutions.	A II	M II		**Examples:** • Within which number system can $x^2 = -2$ be solved? Explain how you know. • Solve $x^2 + 2x + 2 = 0$ over the complex numbers. • Find all solutions of $2x^2 + 5 = 2x$ and express them in the form $a + bi$.
HS.N-CN.8. Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.*	+	+	*HS.MP.7.* Look for and make use of structure.	
HS.N-CN.9. Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Connection: *11-12.WHST.1c*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.7.* Look for and make use of structure.	**Examples:** • How many zeros does $-2x^2 + 3x - 8$ have? Find all the zeros and explain, orally or in written format, your answer in terms of the Fundamental Theorem of Algebra. • How many complex zeros does the following polynomial have? How do you know? $p(x) = (x^2 - 3)(x^2 + 2)(x - 3)(2x - 1)$

Number and Quantity: Vector and Matrix Quantities (N-VM)

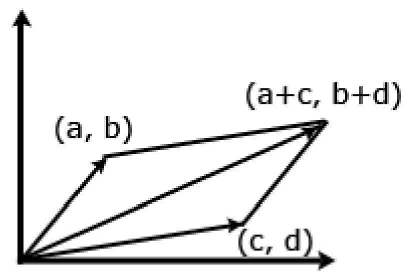
Represent and model with vector quantities

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.1. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., *v*, |*v*|, ||*v*||, *v*).	+	+	*HS.MP.4.* Model with mathematics.	
HS.N-VM.2. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	+	+	*HS.MP.2.* Reason abstractly and quantitatively.	

HS.N-VM.3. Solve problems involving velocity and other quantities that can be represented by vectors.
 	
Connections: *11-12.RST.9; SCHS-S5C2-01*; *SCHS-S5C2-02*; *SCHS-S5C2-06*; *11-12.WHST.2d*
	+	+	*HS.MP.1.* Make sense of problems and persevere in solving them.
			
			HS.MP.2. Reason abstractly and quantitatively.
			
			HS.MP.4. Model with mathematics.
			
			HS.MP.5. Use appropriate tools strategically.
			
			HS.MP.6. Attend to precision.
				Examples:
				• A motorboat traveling from one shore to the other at a rate of 5 m/s east encounters a current flowing at a rate of 3.5 m/s north.
				  ○ What is the resultant velocity?
				  ○ If the width of the river is 60 meters wide, then how much time does it take the boat to travel to the opposite shore?
				  ○ What distance downstream does the boat reach the opposite shore?
				• A ship sails 12 hours at a speed of 15 knots (nautical miles per hour) at a heading of 68° north of east. It then turns to a heading of 75° north of east and travels for 5 hours at 8 knots. Find its position north and east of its starting point.
				(For this problem, assume the earth is flat.)
				• The solution may require an explanation, orally or in written form, that includes understanding of velocity and other relevant quantities.

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on vectors

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.4. Add and subtract vectors.	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Addition of vectors is used to determine the resultant of two given vectors. This can be done by lining up the vectors end to end, adding the components, or using the parallelogram rule. Students may use applets to help them visualize operations of vectors given in rectangular or polar form.  **Example:** • Given two vectors u and v, can the magnitude of the resultant be found by adding the magnitude of each vector? Use an example to illustrate your explanation. • If $u = \langle -2,\ -8 \rangle$ and $v = \langle 2,\ 8 \rangle$, find *u* + *v*, *u* + (-*v*), and *u* – *v*. Explain the relationship between *u* + (-*v*) and *u* – *v* in terms of the vector components. • A plane is flying due east at an average speed of 500 miles per hour. There is a crosswind from the south at 60 miles per hour. What is the magnitude and direction of the resultant?
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	+	+		
b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	+	+		
c. Understand vector subtraction *v* – *w* as *v* + (–*w*), where –*w* is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. Connection: *ETHS-S6C1-03*	+	+		

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on vectors *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.5. Multiply a vector by a scalar.	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	The result of multiplying a vector *v* by a positive scalar *c* is a vector in the same direction as *v* with a magnitude of *cv*. If *c* is negative, then the direction of *v* is reversed by scalar multiplication. Students will represent scalar multiplication graphically and component-wise. Students may use applets to help them visualize operations of vectors given in rectangular or polar form. **Example:** • Given $u = \langle 2, 4 \rangle$, write the components and draw the vectors for *u*, 2*u*, ½*u*, and –*u*. How are the vectors related?
a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.	+	+		
b. Compute the magnitude of a scalar multiple *cv* using $\|\|cv\|\| = \|c\|v$. Compute the direction of ***cv*** knowing that when $\|c\|v \neq 0$, the direction of *cv* is either along *v* (for *c* > 0) or against *v* (for *c* < 0). Connection: *ETHS-S6C1-03*	+	+		

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on matrices and use matrices in applications

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.6. Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. Connections: *9-10.RST.7*; *9-10.WHST.2f*; *11-12.RST.9*; *11-12.WHST.2e* ; *ETHS-S6C2-03*;	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. The adjacency matrix of a simple graph is a matrix with rows and columns labeled by graph vertices, with a 1 or a 0 in position (v_i, v_j) according to whether v_i and v_j are adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. **Example:** • Write an inventory matrix for the following situation. A teacher is buying supplies for two art classes. For class 1, the teacher buys 24 tubes of paint, 12 brushes, and 17 canvases. For class 2, the teacher buys 20 tubes of paint, 14 brushes and 15 canvases. Next year, she has 3 times as many students in each class. What affect does this have on the amount of supplies? Solution: **Year 1** $\begin{array}{ll} & \begin{array}{ccc} P & B & C \end{array} \\ \begin{array}{l} \text{Class 1} \\ \text{Class 2} \end{array} & \begin{bmatrix} 24 & 12 & 17 \\ 20 & 14 & 15 \end{bmatrix} \end{array}$ **Year 2** $\begin{array}{ll} & \begin{array}{ccc} P & B & C \end{array} \\ \begin{array}{l} \text{Class 1} \\ \text{Class 2} \end{array} & \begin{bmatrix} 72 & 36 & 51 \\ 60 & 42 & 45 \end{bmatrix} \end{array}$

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on matrices and use matrices in applications *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. Connections: *9-10.RST.3*; *ETHS-S6C2-03*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. **Example:** • $-3\begin{bmatrix} -7 & 19 & 15 \\ 41 & -63 & 20 \\ 2 & 0 & -8 \end{bmatrix}$ • The following is an inventory matrix for Company A's jellybean, lollipop, and gum flavors. The price per unit is \$0.03 for jelly beans, gum, and lollipops. Determine the gross profit for each flavor and for the entire lot. $\begin{array}{cc} & \begin{array}{ccccccc} F1 & F2 & F3 & F4 & F5 & F6 & F7 \end{array} \\ \begin{array}{c} C1 \\ C2 \\ C3 \end{array} & \begin{bmatrix} 327 & 818 & 465 & 211 & 127 & 134 & 705 \\ 513 & 222 & 312 & 446 & 645 & 671 & 101 \\ 878 & 901 & 51 & 156 & 711 & 423 & 344 \end{bmatrix} \end{array}$ C1 = Jelly beans C2 = Lollipops C3 = Gum F1 = Vanilla F2 = Banana F3 = Strawberry F4 = Tangerine F5 = Coconut F6 = Mint F7 = Licorice

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on matrices and use matrices in applications *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.8. Add, subtract, and multiply matrices of appropriate dimensions. Connections: *9-10.RST.3*; *ETHS-S6C2-03*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. **Example:** • Find $2A - B + C$ and $A \bullet B$ given Matrices *A*, *B* and *C* below. Matrix *A* $\begin{bmatrix} -7 & 19 & 15 \\ 41 & -63 & 20 \\ 2 & 0 & -8 \end{bmatrix}$ Matrix *B* $\begin{bmatrix} 23 & 18 & 55 \\ -18 & -47 & 11 \\ 39 & -6 & -8 \end{bmatrix}$ Matrix *C* $\begin{bmatrix} -4 & 7 & 12 \\ 51 & 9 & 80 \\ 13 & 72 & 8 \end{bmatrix}$
HS.N-VM.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. Connections: *ETHS-S6C2-03*; *9-10.WHST.1e*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.6.* Attend to precision.	Students may use graphing calculators and spreadsheets to create and perform operations on matrices. **Example:** • Given $A = \begin{bmatrix} -1 & 3 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -4 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} 6 & -2 \\ 9 & 7 \end{bmatrix}$; determine if the following statements are true: ○ $AB = BA$ ○ $(AB)C = A(BC)$
HS.N-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.6.* Attend to precision.	

Number and Quantity: Vector and Matrix Quantities (N-VM)

Perform operations on matrices and use matrices in applications *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.N-VM.11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. Connections: *ETHS-S6C1-03*; *11-12.WHST.1a*	+	+	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	A matrix is a two dimensional array with rows and columns; a vector is a one dimensional array that is either one row or one column of the matrix. Students will use matrices to transform geometric objects in the coordinate plane. Students may demonstrate transformations using dynamic geometry programs or applets. They will explain the relationship between the ordered pair representation of a vector and its graphical representation.
HS.N-VM.12. Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. Connection: *ETHS-S6C1-03*	+	+	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students should be able to utilize matrix multiplication to perform reflections, rotations and dilations, and find the area of a parallelogram. Students may demonstrate these relationships using dynamic geometry programs or applets.

High School: Algebra Overview

Seeing Structure in Expressions (A-SSE)

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions (A-APR)

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations (A-CED)

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities (A-REI)

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Algebra

Expressions

An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and Inequalities

An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 – 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling

Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra: Seeing Structure in Expressions (A-SSE)

Interpret the structure of expressions

<u>*Standards*</u> *Students are expected to:*	<u>*TRAD*</u>	<u>*INT*</u>	<u>*Mathematical Practices*</u>	<u>*Explanations and Examples*</u>
HS.A-SSE.1. Interpret expressions that represent a quantity in terms of its context.	A I ★	M I M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.7.* Look for and make use of structure.	Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context.
a. Interpret parts of an expression, such as terms, factors, and coefficients. Connection: *9-10.RST.4*	A I ★	M I ★		
b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	A I ★	M I M II ★		
HS.A-SSE.2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	A I A II	M II M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	Students should extract the greatest common factor (whether a constant, a variable, or a combination of each). If the remaining expression is quadratic, students should factor the expression further. **Example:** • Factor $x^3 - 2x^2 - 35x$

Algebra: Seeing Structure in Expressions (A-SSE)

Write expressions in equivalent forms to solve problems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-SSE.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Connections: *9-10.WHST.1c*; *11-12.WHST.1c*	A I A II ★	M I M II	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively.	Students will use the properties of operations to create equivalent expressions. **Examples:** • Express $2(x^3 - 3x^2 + x - 6) - (x - 3)(x + 4)$ in factored form and use your answer to say for what values of x the expression is zero. • Write the expression below as constant times a power of x and use your answer to decide whether the expression gets larger or smaller as x gets larger. ○ $\dfrac{(2x^3)^2(3x^4)}{(x^2)^3}$
a. Factor a quadratic expression to reveal the zeros of the function it defines.	A I ★	M II ★	*HS.MP.4.* Model with mathematics.	
b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	A I ★	M II ★	*HS.MP.7.* Look for and make use of structure.	
c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	A I A II ★	M I ★		

Algebra: Seeing Structure in Expressions (A-SSE)

Write expressions in equivalent forms to solve problems *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* Connection: *11-12.RST.4*	A II ★	M III ★	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.7.* Look for and make use of structure.	**Example:** • In February, the Bezanson family starts saving for a trip to Australia in September. The Bezanson's expect their vacation to cost $5375. They start with $525. Each month they plan to deposit 20% more than the previous month. Will they have enough money for their trip?

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

Perform arithmetic operations on polynomials

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Connection: *9-10.RST.4*	A I	M II	*HS.MP.8.* Look for regularity in repeated reasoning.	

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

Understand the relationship between zeros and factors of polynomials

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-APR.2. Know and apply the Remainder Theorem: For a polynomial *p*(*x*) and a number *a*, the remainder on division by *x* – *a* is *p*(*a*), so *p*(*a*) = 0 if and only if (*x* – *a*) is a factor of *p*(*x*).	A II	M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others.	The Remainder theorem says that if a polynomial *p*(*x*) is divided by *x* – *a*, then the remainder is the constant *p*(*a*). That is, $p(x)=q(x)(x-a)+p(a)$. So if p(a) = 0 then p(x) = q(x)(x-a). • Let $p(x)=x^5-3x^4+8x^2-9x+30$. Evaluate p(-2). What does your answer tell you about the factors of p(x)? [Answer: p(-2) = 0 so x+2 is a factor.]
HS.A-APR.3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	A I A II	M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Graphing calculators or programs can be used to generate graphs of polynomial functions. **Example:** • Factor the expression $x^3+4x^2-59x-126$ and explain how your answer can be used to solve the equation $x^3+4x^2-59x-126=0$. Explain why the solutions to this equation are the same as the x-intercepts of the graph of the function $f(x)=x^3+4x^2-59x-126$.

Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

Use polynomial identities to solve problems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-APR.4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	A II	M III	*HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	**Examples:** Use the distributive law to explain why $x^2 - y^2 = (x - y)(x + y)$ for any two numbers x and y. Derive the identity $(x - y)^2 = x^2 - 2xy + y^2$ from $(x + y)^2 = x^2 + 2xy + y^2$ by replacing y by $-y$. Use an identity to explain the pattern $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ [Answer: $(n + 1)^2 - n^2 = 2n + 1$ for any whole number n.]
HS.A-APR.5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. (The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	**Examples:** • Use Pascal's Triangle to expand the expression $(2x - 1)^4$. • Find the middle term in the expansion of $(x^2 + 2)^{18}$. 1 1 1 1 2 1 1 3 3 1 ← $(x+1)^3 = x^3+3x^2+3x+1$ 1 4 6 4 1 ↑ ↑ ↑ ↑ ↑ ${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$

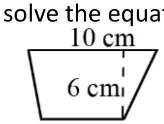
Algebra: Arithmetic with Polynomials and Rational Expressions (A-APR)

Rewrite rational expressions

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-APR.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	A II	M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	The polynomial $q(x)$ is called the quotient and the polynomial $r(x)$ is called the remainder. Expressing a rational expression in this form allows one to see different properties of the graph, such as horizontal asymptotes. **Examples:** • Find the quotient and remainder for the rational expression $\frac{x^3-3x^2+x-6}{x^2+2}$ and use them to write the expression in a different form. • Express $f(x)=\frac{2x+1}{x-1}$ in a form that reveals the horizontal asymptote of its graph. [Answer: $f(x)=\frac{2x+1}{x-1}=\frac{2(x-1)+3}{x-1}=2+\frac{3}{x-1}$, so the horizontal asymptote is $y = 2$.]
HS.A-APR.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	+	+	*HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	**Examples:** • Use the formula for the sum of two fractions to explain why the sum of two rational expressions is another rational expression. • Express $\frac{1}{x^2+1}-\frac{1}{x^2-1}$ in the form $a(x)/b(x)$, where $a(x)$ and $b(x)$ are polynomials.

Algebra: Creating Equations ★ (A-CED)

Create equations that describe numbers or relationships

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-CED.1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	A I A II ★	M I M II M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth. **Examples:** • Given that the following trapezoid has area 54 cm^2, set up an equation to find the length of the base, and solve the equation.  • Lava coming from the eruption of a volcano follows a parabolic path. The height *h* in feet of a piece of lava *t* seconds after it is ejected from the volcano is given by $h(t) = -t^2 + 16t + 936$. After how many seconds does the lava reach its maximum height of 1000 feet?
HS.A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	A I ★	M I M II M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	

Algebra: Creating Equations ★ (A-CED)

Create equations that describe numbers or relationships *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	A I ★	M I ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	**Example:** • A club is selling hats and jackets as a fundraiser. Their budget is $1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs $5 and each jacket costs $8. ○ Write a system of inequalities to represent the situation. ○ Graph the inequalities. ○ If the club buys 150 hats and 100 jackets, will the conditions be satisfied? ○ What is the maximum number of jackets they can buy and still meet the conditions?
HS.A-CED.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*	A I ★	M I M II ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	**Examples:** • The Pythagorean Theorem expresses the relation between the legs a and b of a right triangle and its hypotenuse c with the equation $a^2 + b^2 = c^2$. ○ Why might the theorem need to be solved for c? ○ Solve the equation for c and write a problem situation where this form of the equation might be useful. ○ Solve $V = \frac{4}{3}\pi r^3$ for radius *r*. • Motion can be described by the formula below, where *t* = time elapsed, *u*=initial velocity, *a* = acceleration, and *s* = distance traveled $s = ut+½at^2$ ○ Why might the equation need to be rewritten in terms of *a*? ○ Rewrite the equation in terms of *a*.

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning

Standards *Students are expected to:*	TRAD	INT	Mathematical Practices	Explanations and Examples
HS.A-REI.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	A I A II	M II M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.7.* Look for and make use of structure.	Properties of operations can be used to change expressions on either side of the equation to equivalent expressions. In addition, adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with the same solutions. Other operations, such as squaring both sides, may produce equations that have extraneous solutions. **Examples:** • Explain why the equation $x/2 + 7/3 = 5$ has the same solutions as the equation $3x + 14 = 30$. Does this mean that $x/2 + 7/3$ is equal to $3x + 14$? • Show that $x = 2$ and $x = -3$ are solutions to the equation $x^2 + x = 6$. Write the equation in a form that shows these are the only solutions, explaining each step in your reasoning.
HS.A-REI.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	A II	M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. HS.*MP.7.* Look for and make use of structure.	**Examples:** • $\sqrt{x+2} = 5$ • $\frac{7}{8}\sqrt{2x-5} = 21$ • $\frac{x+2}{x+3} = 2$ • $\sqrt{3x-7} = -4$

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve equations and inequalities in one variable

<table>
<tr><th><u>Standards</u>
Students are expected to:</th><th><u>TRAD</u></th><th><u>INT</u></th><th><u>Mathematical Practices</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>HS.A-REI.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td><td>A I</td><td>M I</td><td>HS.MP.2. Reason abstractly and quantitatively.
HS.MP.7. Look for and make use of structure.
HS.MP.8. Look for and express regularity in repeated reasoning.</td><td>Examples:
• $-\frac{7}{3}y-8=111$
• $3x > 9$
• $ax + 7 = 12$
• $\frac{3+x}{7}=\frac{x-9}{4}$
• Solve for x: $2/3x + 9 < 18$</td></tr>
<tr><td>HS.A-REI.4. Solve quadratic equations in one variable.</td><td>A I
A II</td><td>M II</td><td rowspan="3">HS.MP.2. Reason abstractly and quantitatively.
HS.MP.7. Look for and make use of structure.
HS.MP.8. Look for and express regularity in repeated reasoning.</td><td rowspan="3">Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of
$y = ax^2 + bx + c$.
<table>
<tr><th>Value of Discriminant</th><th>Nature of Roots</th><th>Nature of Graph</th></tr>
<tr><td>$b^2 – 4ac = 0$</td><td>1 real roots</td><td>intersects x-axis once</td></tr>
<tr><td>$b^2 – 4ac > 0$</td><td>2 real roots</td><td>intersects x-axis twice</td></tr>
<tr><td>$b^2 – 4ac < 0$</td><td>2 complex roots</td><td>does not intersect x-axis</td></tr>
</table>
Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have? Find all solutions of the equation.
• What is the nature of the roots of $x^2 + 6x + 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?</td></tr>
<tr><td>a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</td><td>A I</td><td>M II</td></tr>
<tr><td>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.</td><td>A I
A II</td><td>M II</td></tr>
</table>

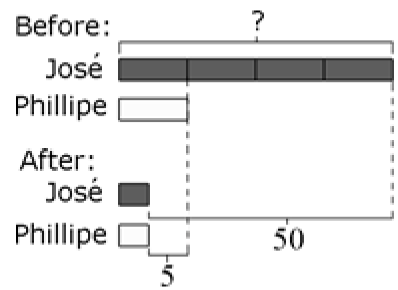
Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve systems of equations

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	*A I*	M I	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others.	**Example:** Given that the sum of two numbers is 10 and their difference is 4, what are the numbers? Explain how your answer can be deduced from the fact that they two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$.

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve systems of equations *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Connection: *ETHS-S6C2-03*	A I A II	M I	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations. **Examples:** • José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards. Write a system to describe the situation and solve the system.  • Solve the system of equations: $x + y = 11$ and $3x - y = 5$. • Use a second method to check your answer. • Solve the system of equations: • $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$. • The opera theater contains 1,200 seats, with three different prices. The seats cost $45 dollars per seat, $50 per seat, and $60 per seat. The opera needs to gross $63,750 on seat sales. There are twice as many $60 seats as $45 seats. How many seats in each level need to be sold?

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Solve systems of equations *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* *$y = -3x$ and the circle $x^2 + y^2 = 3$.*	A II	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	**Example:** • Two friends are driving to the Grand Canyon in separate cars. Suzette has been there before and knows the way but Andrea does not. During the trip Andrea gets ahead of Suzette and pulls over to wait for her. Suzette is traveling at a constant rate of 65 miles per hour. Andrea sees Suzette drive past. To catch up, Andrea accelerates at a constant rate. The distance in miles (*d*) that her car travels as a function of time in hours (*t*) since Suzette's car passed is given by $d = 3500t^2$. Write and solve a system of equations to determine how long it takes for Andrea to catch up with Suzette.
HS.A-REI.8. Represent a system of linear equations as a single matrix equation in a vector variable.	+	+		**Example:** • Write the system $\begin{cases} -b + 2c = 4 \\ a + b - c = 0 \\ 2a + 3c = 11 \end{cases}$ as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

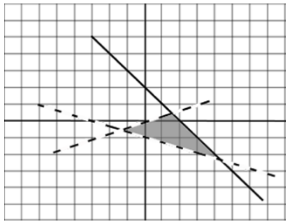
Solve systems of equations *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.9. Find the inverse of a matrix if it exists, and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). Connection: *ETHS-S6C2-03*	+	+	*HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students will perform multiplication, addition, subtraction, and scalar multiplication of matrices. They will use the inverse of a matrix to solve a matrix equation. Students may use graphing calculators, programs, or applets to model and find solutions for systems of equations. **Example:** • Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix. $\begin{cases} 5x+2y=4 \\ 3x+2y=0 \end{cases}$ Solution: Matrix $A=\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$ Matrix $X=\begin{bmatrix} x \\ y \end{bmatrix}$ Matrix $B=\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ Matrix $A^{-1}=\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}$ $X=A^{-1}B$ $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}=\begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Algebra: Reasoning with Equations and Inequalities ★ (A-REI)

Represent and solve equations and inequalities graphically

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	A I	M I	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics.	**Example:** • Which of the following points is on the circle with equation $(x-1)^2+(y+2)^2=5$? (a) (1, -2) (b) (2, 2) (c) (3, -1) (d) (3, 4)
HS.A-REI.11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Connection: *ETHS-S6C2-03*	A I A II ★	M I M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions. **Example:** • Given the following equations determine the *x* value that results in an equal output for both functions. $f(x) = 3x - 2$ $g(x) = (x+3)^2 - 1$

Algebra: Reasoning with Equations and Inequalities ★ (A-REI) **Represent and solve equations and inequalities graphically** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.A-REI.12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	A I	M I	*HS.MP.4.* Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Students may use graphing calculators, programs, or applets to model and find solutions for inequalities or systems of inequalities. **Examples:** • Graph the solution: $y \leq 2x + 3$. • A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set. • Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system. $\begin{cases} x - 3y > 0 \\ x + y \leq 2 \\ x + 3y > -3 \end{cases}$ Solution:  (3, 2) is not an element of the solution set (graphically or by substitution).

High School: Function Overview

Interpreting Functions (F-IF)

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions (F-BF)

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models (F-LE)

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions (F-TF)

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions: Interpreting Functions (F-IF)

Understand the concept of a function and use of function notation

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*).	A I	M I	*HS.MP.2.* Reason abstractly and quantitatively.	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain.
HS.F-IF.2. Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Connection: *9-10.RST.4*	A I	M I	*HS.MP.2.* Reason abstractly and quantitatively.	The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain. **Examples:** • If $f(x)=x^2+4x-12$, find $f(2)$. • Let $f(x)=2(x+3)^2$. Find $f(3)$, $f(-\frac{1}{2})$, $f(a)$, and $f(a-h)$ • If P(*t*) is the population of Tucson *t* years after 2000, interpret the statements P(0) = 487,000 and P(10)-P(9) = 5,900.
HS.F-IF.3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	A I A II	M I	*HS.MP.8.* Look for and express regularity in repeated reasoning.	

Functions: Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of context

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*

HS.F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
Connections: *ETHS-S6C2.03*; *9-10.RST.7*; *11-12.RST.7*
	A I
	A II
	★
		M I
		M II
		M III
		★
			HS.MP.2. Reason abstractly and quantitatively.
			HS.MP.4. Model with mathematics.
			HS.MP.5. Use appropriate tools strategically.
			HS.MP.6. Attend to precision.
				Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.
				Examples:
				• A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where t is measured in seconds and h is height above the ground measured in feet.
				  ○ What is a reasonable domain restriction for t in this context?
				  ○ Determine the height of the rocket two seconds after it was launched.
				  ○ Determine the maximum height obtained by the rocket.
				  ○ Determine the time when the rocket is 100 feet above the ground.
				  ○ Determine the time at which the rocket hits the ground.
				  ○ How would you refine your answer to the first question based on your response to the second and fifth questions?
				• Compare the graphs of $y = 3x^2$ and $y = 3x^3$.
				• Let $R(x) = \frac{2}{\sqrt{x-2}}$. Find the domain of $R(x)$. Also find the range, zeros, and asymptotes of $R(x)$.
				• Let $f(x) = 5x^3 - x^2 - 5x + 1$. Graph the function and identify end behavior and any intervals of constancy, increase, and decrease.
				• It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn't rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.

Functions: Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of context *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* Connection: *9-10.WHST.2f*	A I ★	M I M II ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision.	Students may explain orally, or in written format, the existing relationships.

Functions: Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of context *continued*

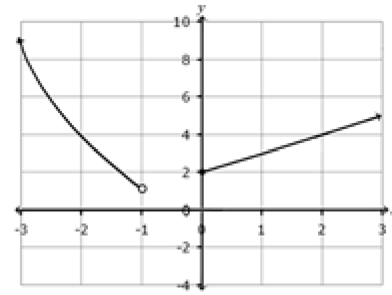
Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Connections: *ETHS-S1C2-01*; *9-10.RST.3*	A I A II ★	M I M II M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	The average rate of change of a function $y = f(x)$ over an interval [a,b] is $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$. In addition to finding average rates of change from functions given symbolically, graphically, or in a table, Students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation. **Examples:** • Use the following table to find the average rate of change of *g* over the intervals [-2, -1] and [0,2]: (see table below) • The table below shows the elapsed time when two different cars pass a 10, 20, 30, 40 and 50 meter mark on a test track. ○ For car 1, what is the average velocity (change in distance divided by change in time) between the 0 and 10 meter mark? Between the 0 and 50 meter mark? Between the 20 and 30 meter mark? Analyze the data to describe the motion of car 1. ○ How does the velocity of car 1 compare to that of car 2? (see table below)

x	*g(x)*
-2	2
-1	-1
0	-4
2	-10

	Car 1	*Car 2*
d	***t***	***t***
10	4.472	1.742
20	6.325	2.899
30	7.746	3.831
40	8.944	4.633
50	10	5.348

Functions: Interpreting Functions (F-IF)

Analyze functions using different representation

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	A I A II + ★	M I M II M III + ★	*HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Describe key characteristics of the graph of *f(x)* = |*x* – 3| + 5. • Sketch the graph and identify the key characteristics of the function described below. $F(x)=\begin{cases} x+2 \text{ for } x\geq 0 \\ -x^2 \text{ for } x<-1 \end{cases}$  • Graph the function $f(x) = 2^x$ by creating a table of values. Identify the key characteristics of the graph. • Graph $f(x) = 2 \tan x – 1$. Describe its domain, range, intercepts, and asymptotes. • Draw the graph of $f(x) = \sin x$ and $f(x) = \cos x$. What are the similarities and differences between the two graphs?
a. Graph linear and quadratic functions and show intercepts, maxima, and minima. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	A I ★	M I M II ★		
b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	A I ★	M II ★		
c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03* *Continued on next page*	A II ★	M III ★		

Functions: Interpreting Functions (F-IF)

Analyze functions using different representation *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.7. continued d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	+ ★	+ ★		
e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	A II ★	M II M III ★		

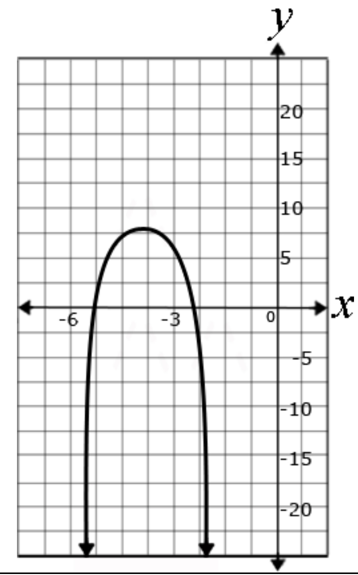
Functions: Interpreting Functions (F-IF)

Analyze functions using different representation *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Connection: *11-12.RST.7*	A I A II	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.7.* Look for and make use of structure.	
a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Connection: *11-12.RST.7*	A I	M II		
b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* Connection: *11-12.RST.7*	A II	M II		

Functions: Interpreting Functions (F-IF)

Analyze functions using different representation *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-IF.9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03;9-10.RST.7*	A I A II	M I M II M III	*HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	**Example:** • Examine the functions below. Which function has the larger maximum? How do you know? $f(x) = -2x^2 - 8x + 20$ 

Functions: Building Functions (F-BF)

Build a function that models a relationship between two quantities

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-BF.1. Write a function that describes a relationship between two quantities. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	A I A II + ★	M I M II + ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students will analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function’s description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation. • A cup of coffee is initially at a temperature of 93º F. The difference between its temperature and the room temperature of 68º F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time. • The radius of a circular oil slick after *t* hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \le t \le 10$. Find the area of the oil slick as a function of time.
a. Determine an explicit expression, a recursive process, or steps for calculation from a context. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*; *9-10.RST.7*; *11-12.RST.7*	A I A II ★	M I M II ★		
b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03* *Continued on next page*	A II ★	M II ★		

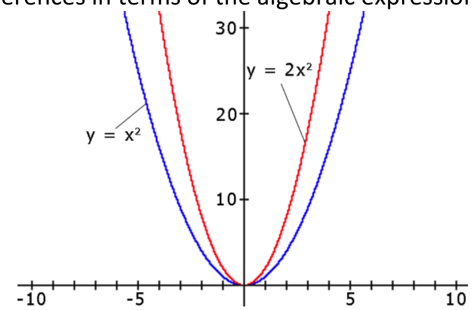
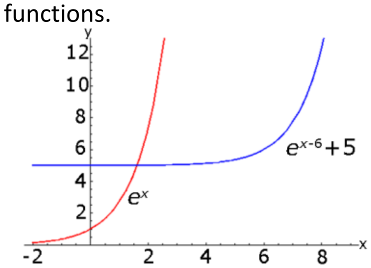
Functions: Building Functions (F-BF)

Build a function that models a relationship between two quantities *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-BF.1. *continued* c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*	+ ★	+ ★		
HS.F-BF.2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	A II ★	M I ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.8.* Look for and express regularity in repeated reasoning.	An explicit rule for the *n*th term of a sequence gives a_n as an expression in the term's position *n*; a recursive rule gives the first term of a sequence, and a recursive equation relates a_n to the preceding term(s). Both methods of presenting a sequence describe a_n as a function of *n*. **Examples:** • Generate the 5^{th}-11^{th} terms of a sequence if $A_1 = 2$ and $A_{(n+1)} = (A_n)^2 - 1$ • Use the formula: $A_n = A_1 + d(n - 1)$ where d is the common difference to generate a sequence whose first three terms are: -7, -4, and -1. • There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of the year. Find the population in five years. Also, find the long-term population. • Given the formula $A_n = 2n - 1$, find the 17^{th} term of the sequence. What is the 9^{th} term in the sequence 3, 5, 7, 9, …? • Given $a_1 = 4$ and $a_n = a_{n-1} + 3$, write the explicit formula.

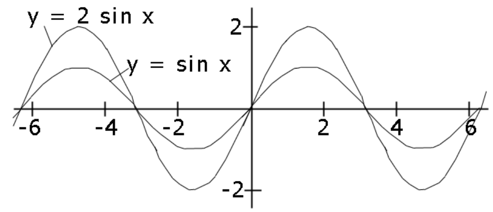
Functions: Building Functions (F-BF)

Build new functions from existing functions

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-BF.3. Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k f*(*x*), *f*(*kx*), and *f*(*x* + *k*) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* Connections: *ETHS-S6C2-03*; *11-12.WHST.2e*	A I A II	M II M III	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students will apply transformations to functions and recognize functions as even and odd. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions. **Examples:** • Is $f(x) = x^3 - 3x^2 + 2x + 1$ even, odd, or neither? Explain your answer orally or in written format. • Compare the shape and position of the graphs of $f(x) = x^2$ and $g(x) = 2x^2$, and explain the differences in terms of the algebraic expressions for the functions.  • Describe effect of varying the parameters *a, h,* and *k* have on the shape and position of the graph of $f(x) = a(x-h)^2 + k$. • Compare the shape and position of the graphs of $f(x) = e^x$ to $g(x) = e^{x-6} + 5$, and explain the differences, orally or in written format, in terms of the algebraic expressions for the functions.  *Continued on next page*

Functions: Building Functions (F-BF)

Build new functions from existing functions *continued*

<u>*Standards*</u> *Students are expected to:*	<u>*TRAD*</u>	<u>*INT*</u>	<u>*Mathematical Practices*</u>	<u>*Explanations and Examples*</u>
HS.F-BF.3 *continued*				• Describe the effect of varying the parameters *a, h,* and *k* on the shape and position of the graph f(*x*) = $ab^{(x+h)}$ + k., orally or in written format. What effect do values between 0 and 1 have? What effect do negative values have? • Compare the shape and position of the graphs of *y* = sin *x* to *y* = 2 sin *x.* 
HS.F-BF.4 Find inverse functions. Connection: *ETHS-S6C2-03*	A II +	M II +	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions. **Examples:** • For the function *h*(*x*) = $(x-2)^3$, defined on the domain of all real numbers, find the inverse function if it exists or explain why it doesn't exist. • Graph h(*x*) and $h^{-1}(x)$ and explain how they relate to each other graphically. • Find a domain for f(*x*) = $3x^2 + 12x - 8$ on which it has an inverse. Explain why it is necessary to restrict the domain of the function.
a. Solve an equation of the form *f*(*x*) = *c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	A II	M III		
b. Verify by composition that one function is the inverse of another.	+	+		
c. Read values of an inverse function from a graph or a table, given that the function has an inverse.	+	+		
d. Produce an invertible function from a non-invertible function by restricting the domain.	+	+		

Functions: Building Functions (F-BF)

Build new functions from existing functions *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-BF.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. Connection: *ETHS-S6C2-03*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to solve problems involving logarithms and exponents. **Example:** • Find the inverse of $f(x) = 3(10)^{2x}$.

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions. Connections: *ETHS-S6C2-03*; *SSHS-S5C5-03*	A I ★	M I ★	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions. **Examples:** • A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in Plan 1? Plan 2? Plan 3? 1. $59.95/month for 700 minutes and $0.25 for each additional minute, 2. $39.95/month for 400 minutes and $0.15 for each additional minute, and 3. $89.95/month for 1,400 minutes and $0.05 for each additional minute. • A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit? Students can investigate functions and graphs modeling different situations involving simple and compound interest. Students can compare interest rates with different periods of compounding (monthly, daily) and compare them with the corresponding annual percentage rate. Spreadsheets and applets can be used to explore and model different interest rates and loan terms. Students can use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions. • A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal? • Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has? ○ Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest. ○ Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually. • Calculate the future value of a given amount of money, with and without technology. • Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.
a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. Connection: *11-12.WHST.1a-1e*	A I ★	M I ★		
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. Connection: *11-12.RST.4*	A I ★	M I ★		

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems *continued*

<table>
<tr><th><u>Standards</u>
Students are expected to:</th><th><u>TRAD</u></th><th><u>INT</u></th><th><u>Mathematical Practices</u></th><th><u>Explanations and Examples</u></th></tr>
<tr><td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4</td><td>A I
★</td><td>M I
★</td><td></td><td></td></tr>
<tr><td>HS.F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
Connections: ETHS-S6C1-03; ETHS-S6C2-03; 11-12.RST.4; SSHS-S5C5-03</td><td>A I
A II
★</td><td>M I
★</td><td>HS.MP.4. Model with mathematics.
HS.MP.8. Look for and express regularity in repeated reasoning.</td><td>Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.
Examples:
Determine an exponential function of the form $f(x) = ab^x$ using data points from the table. Graph the function and identify the key characteristics of the graph.<table><tr><th>x</th><th>f(x)</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>27</td></tr></table>
Sara's starting salary is $32,500. Each year she receives a $700 raise. Write a sequence in explicit form to describe the situation.</td></tr>
<tr><td>HS.F-LE.3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.</td><td>A I
★</td><td>M I
★</td><td>HS.MP.2. Reason abstractly and quantitatively.</td><td>Example:
Contrast the growth of the $f(x)=x^3$ and $f(x)=3^x$.</td></tr>
</table>

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Construct and compare linear, quadratic, and exponential models and solve problems *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-LE.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03*; *11-12.RST.3*	A II ★	M III ★	*HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to analyze exponential models and evaluate logarithms. Example: • Solve $200\, e^{0.04t} = 450$ for t. Solution: We first isolate the exponential part by dividing both sides of the equation by 200. $e^{0.04t} = 2.25$ Now we take the natural logarithm of both sides. $ln\, e^{0.04t} = ln\ 2.25$ The left hand side simplifies to $0.04t$, by logarithmic identity 1. $0.04t = ln\ 2.25$ Lastly, divide both sides by 0.04 $t = ln\ (2.25) / 0.04$ $t \approx 20.3$

Functions: Linear, Quadratic, and Exponential Models ★ (F-LE)

Interpret expressions for functions in terms of the situation they model

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-LE.5. Interpret the parameters in a linear or exponential function in terms of a context. Connections: *ETHS-S6C1-03*; *ETHS-S6C2-03;SSHS-S5C5-03*; *11-12.WHST.2e*	A I A II ★	M I ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions. Example: A function of the form $f(n) = P(1 + r)^n$ is used to model the amount of money in a savings account that earns 5% interest, compounded annually, where n is the number of years since the initial deposit. What is the value of r? What is the meaning of the constant P in terms of the savings account? Explain either orally or in written format.

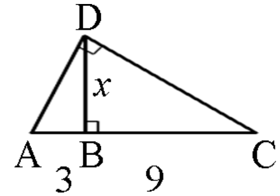
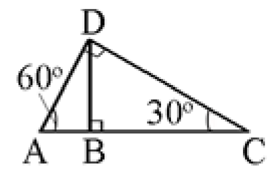
Functions: Trigonometric Functions ★ (F-TF)

Extend the domain of trigonometric functions using the unit circle

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	A II	M III		
HS.F-TF.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. Connections: *ETHS-S1C2-01*; *11-12.WHST.2b*; *11-12.WHST.2e*	A II	M III	*HS.MP.2.* Reason abstractly and quantitatively.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or in written format) their understanding.

Functions: Trigonometric Functions ★ (F-TF)

Extend the domain of trigonometric functions using the unit circle *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.3. Use special triangles to determine geometrically the values of sine, cosine, tangent for π /3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π-*x*, π+*x*, and 2π-*x* in terms of their values for *x*, where *x* is any real number. Connection: *11-12.WHST.2b*	+	+	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	**Examples:** • Evaluate all six trigonometric functions of $\vartheta = \frac{\pi}{3}$. • Evaluate all six trigonometric functions of $\vartheta = 225°$. • Find the value of x in the given triangle where $\overline{AD} \perp \overline{DC}$ and $\overline{AC} \perp \overline{DB}$ $m\angle A = 60°, m\angle C = 30°$. Explain your process for solving the problem including the use of trigonometric ratios as appropriate.  • Find the measure of the missing segment in the given triangle where $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{DB}$, $m\angle A = 60°, m\angle C = 30°, \overline{AC} = 12, \overline{AB} = 3$. Explain (orally or in written format) your process for solving the problem including use of trigonometric ratios as appropriate. 

Functions: Trigonometric Functions ★ (F-TF)

Extend the domain of trigonometric functions using the unit circle *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.4. Use the units circle to explain symmetry (odd and even) and periodicity of trigonometric functions. Connections: *ETHS-S1C2-01*; *11-12.WHST.2c*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use applets and animations to explore the unit circle and trigonometric functions. Students may explain (orally or written format) their understanding of symmetry and periodicity of trigonometric functions.

Functions: Trigonometric Functions ★ (F-TF)

Model periodic phenomena with trigonometric functions

Standards *Students are expected to:*		*Label*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. Connection: *ETHS-S1C2-01*	A II ★	M III ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and periodic phenomena. **Example:** • The temperature of a chemical reaction oscillates between a low of 20° C and a high of 120° C. The temperature is at its lowest point when t = 0 and completes one cycle over a six hour period. a. Sketch the temperature, T, against the elapsed time, t, over a 12 hour period. b. Find the period, amplitude, and the midline of the graph you drew in part a). c. Write a function to represent the relationship between time and temperature. d. What will the temperature of the reaction be 14 hours after it began? *e.* At what point during a 24 hour day will the reaction have a temperature of 60° C?

Functions: Trigonometric Functions ★ (F-TF)

Model periodic phenomena with trigonometric functions *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.6. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. Connections: *ETHS-S1C2-01*; *11-12.WHST.2e*	+	+		Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions. **Examples:** • Identify a domain for the sine function that would permit an inverse function to be constructed. • Describe the behavior of the graph of the sine function over this interval. • Explain (orally or in written format) why the domain cannot be expanded any further.
HS.F-TF.7. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. Connections: *ETHS-S1C2-01; 11-12.WHST.1a*	+ ★	+ ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.5.* Use appropriate tools strategically.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model trigonometric functions and solve trigonometric equations. **Example:** • Two physics students set up an experiment with a spring. In their experiment, a weighted ball attached to the bottom of the spring was pulled downward 6 inches from the rest position. It rose to 6 inches above the rest position and returned to 6 inches below the rest position once every 6 seconds. The equation $h = -6\cos\left(\frac{\pi}{2}t\right)$ accurately models the height above and below the rest position every 6 seconds. Students may explain, orally or in written format, when the weighted ball first will be at a height of 3 inches, 4 inches, and 5 inches above rest position.

Functions: Trigonometric Functions ★ (F-TF)

Prove and apply trigonometric identities

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.F-TF.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. Connection: *11-12.WHST.1a-1e*	A II	M III	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	
HS.F-TF.9. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. Connection: *11-12.WHST.1a-1e*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

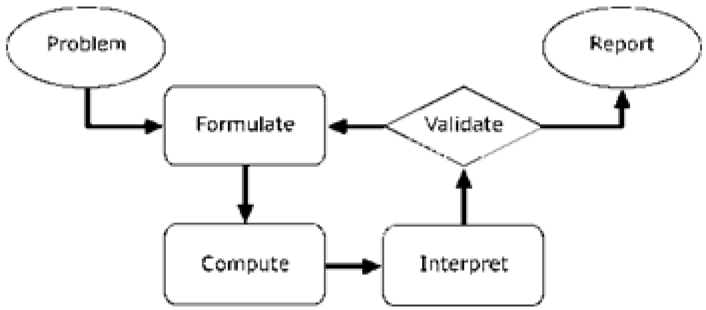
- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.

High School: Modeling (*continued*)

The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.



In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters which are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★).

High School: Geometry Overview

Congruence (G-CO)

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry (G-SRT)

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles (G-C)

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations (G-GPE)

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension (G-GMD)

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry (G-MG)

- Apply geometric concepts in modeling situations

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

High School: Geometry (*continued*)

Connections to Equations

The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry: Congruence (G-CO)

Experiment with transformations in the plane

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-CO.1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. Connection: *9-10.RST.4*	G	M I	*HS.MP.6.* Attend to precision.	
HS.G-CO.2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Connection: *ETHS-S6C1-03*	G	M I	*HS.MP.5.* Use appropriate tools strategically.	Students may use geometry software and/or manipulatives to model and compare transformations.
HS.G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygons, describe the rotations and reflections that carry it onto itself. Connections: *ETHS-S6C1-03*; *9-10.WHST.2c*	G	M I	*HS.MP.3* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometry software and/or manipulatives to model transformations.

Geometry: Congruence (G-CO)

Experiment with transformations in the plane *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-CO.4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Connections: *ETHS-S6C1-03*; *9-10.WHST.4*	G	M I	*HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students may use geometry software and/or manipulatives to model transformations. Students may observe patterns and develop definitions of rotations, reflections, and translations.
HS.G-CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Connections: *ETHS-S6C1-03*; *9-10.WHST.3*	G	M I	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.

Geometry: Congruence (G-CO)

Understand congruence in terms of rigid motions

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-CO.6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Connections: *ETHS-S1C2-01*; *9-10.WHST.1e*	G	M I	*HS.MP.3*. Construct viable arguments and critique the reasoning of others. *HS.MP.5*. Use appropriate tools strategically. *HS.MP.7*. Look for and make use of structure.	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Students may use geometric software to explore the effects of rigid motion on a figure(s).
HS.G-CO.7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Connection: *9-10.WHST.1e*	G	M I	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures. Congruence of triangles Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.
HS.G-CO.8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Connection: *9-10.WHST.1e*	G	MI	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	

Geometry: Congruence (G-CO)

Prove geometric theorems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-CO.9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* Connections: *ETHS-S1C2-01; 9-10.WHST.1a-1e*	G	M I	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about lines and angles.
HS.G-CO.10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* Connections: *ETHS-S1C2-01; 9-10.WHST.1a-1e*	G	M I	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.

Geometry: Congruence (G-CO) **Prove geometric theorems** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-CO.11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* Connection: *9-10.WHST.1a-1e*	**G**	M I	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

Geometry: Congruence (G-CO)

Make geometric constructions

Standards *Students are expected to:*	TRAD	INT	Mathematical Practices	Explanations and Examples
HS.G-CO.12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* Connection: *ETHS-S6C1-03*	G	M III	*HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Students may use geometric software to make geometric constructions. **Examples:** • Construct a triangle given the lengths of two sides and the measure of the angle between the two sides. • Construct the circumcenter of a given triangle.
HS.G-CO.13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Connection: *ETHS-S6C1-03*	G	M III	*HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Students may use geometric software to make geometric constructions.

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.1. Verify experimentally the properties of dilations given by a center and a scale factor: Connections: *ETHS-S1C2-01*; *9-10.WHST.1b*; *9-10.WHST.1e*	G	M II	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.5.* Use appropriate tools strategically.	Dilation is a transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor. Students may use geometric simulation software to model transformations. Students may observe patterns and verify experimentally the properties of dilations.
a. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	G	M II		
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	G	M II		
HS.G-SRT.2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. Connections: *ETHS-S1C2-01*; *9-10.RST.4*; *9-10.WHST.1c*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	A similarity transformation is a rigid motion followed by dilation. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

Understand similarity in terms of similarity transformations *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Connections: *ETHS-S1C2-01*; *9-10.RST.7*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	

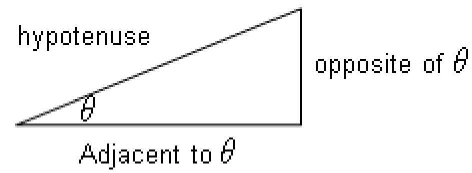
Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

Prove theorems involving similarity

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* Connections: *ETHS-S1C2-01*; *9-10.WHST.1a-1e*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5*. Use appropriate tools strategically.	Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.
HS.G-SRT.5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. Connections: *ETHS-S1C2-01*; *9-10.WHST.1a-1e*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. HS.MP.5. Use appropriate tools strategically.	Similarity postulates include SSS, SAS, and AA. Congruence postulates include SSS, SAS, ASA, AAS, and H-L. Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.

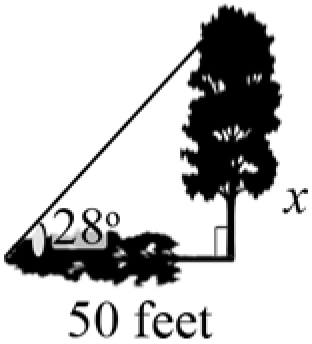
Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

Define trigonometric ratios and solve problems involving right triangles

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Connection: *ETHS-S6C1-03*	G	M II	*HS.MP.6.* Attend to precision. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use applets to explore the range of values of the trigonometric ratios as θ ranges from 0 to 90 degrees.  $\text{sine of } \vartheta = \sin\vartheta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad$ $\text{cosecant of } \vartheta = \csc\vartheta = \frac{\text{hypotenuse}}{\text{opposite}}$ $\text{cosine of } \vartheta = \cos\vartheta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad$ $\text{secant of } \vartheta = \sec\vartheta = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\text{tangent of } \vartheta = \tan\vartheta = \frac{\text{opposite}}{\text{adjacent}}$ $\quad$ $\text{cotangent of } \vartheta = \cot\vartheta = \frac{\text{adjacent}}{\text{opposite}}$
HS.G-SRT.7. Explain and use the relationship between the sine and cosine of complementary angles. Connections: *ETHS-S1C2-01*; *ETHS-S6C1-03*;*9-10.WHST.1c*; *9-10.WHST.1e*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others.	Geometric simulation software, applets, and graphing calculators can be used to explore the relationship between sine and cosine.

Geometry: Similarity, Right Triangles, and Trigonometry (G-SRT)

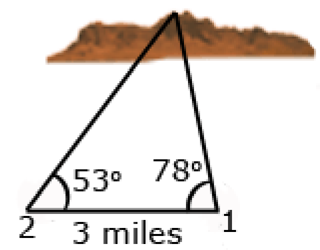
Define trigonometric ratios and solve problems involving right triangles *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Connections: *ETHS-S6C2-03*; *9-10.RST.7*	G ★	M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use graphing calculators or programs, tables, spreadsheets, or computer algebra systems to solve right triangle problems. **Example:** Find the height of a tree to the nearest tenth if the angle of elevation of the sun is 28° and the shadow of the tree is 50 ft. 

Geometry: Circles (G-SRT)

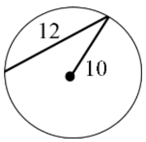
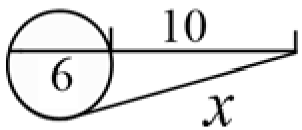
Apply trigonometry to general triangles

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.9. Derive the formula *A* = ½ *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. Connection: *ETHS-S6C1-03*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.7.* Look for and make use of structure.	
HS.G-SRT.10. Prove the Laws of Sines and Cosines and use them to solve problems. Connections: *ETHS-S6C1-03*; *11-12.WHST.1a-1e*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	

Geometry: Circles (G-SRT) **Apply trigonometry to general triangles** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-SRT.11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). Connections: *11-12.WHST.2c; 11-12.WHST.2e*	+	+	HS.MP.1. Make sense of problems and persevere in solving them. HS.MP.4. Model with mathematics.	**Example:** • Tara wants to fix the location of a mountain by taking measurements from two positions 3 miles apart. From the first position, the angle between the mountain and the second position is 78°. From the second position, the angle between the mountain and the first position is 53°. How can Tara determine the distance of the mountain from each position, and what is the distance from each position? 

Geometry: Circles (G-C)

Understand and apply theorems about circles

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-C.1. Prove that all circles are similar. Connections: *ETHS-S1C2-01; 9-10.WHST.1a-1e*	G	M III	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.
HS.G-C.2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* Connections: *9-10.WHST.1c; 11-12.WHST.1c*	G	M III	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	**Examples:** • Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.  • Find the unknown length in the picture below. 
HS.G-C.3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	G	M III	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulation software to make geometric constructions.

Geometry: Circles (G-C)

Understand and apply theorems about circles *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-C.4. Construct a tangent line from a point outside a given circle to the circle. Connection: *ETHS-S6C1-03*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulation software to make geometric constructions.

Geometry: Circles (G-C)

Find arc lengths and areas of sectors of circles

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-C.5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Connections: *ETHS-S1C2-01; 11-12.RST.4*	G	M III	*HS.MP.2* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others.	Students can use geometric simulation software to explore angle and radian measures and derive the formula for the area of a sector.

Geometry: Expressing Geometric Properties with Equations (G-GPE)

Translate between the geometric description and the equation for a conic section

Standards *Students are expected to:*	TRAD	INT	Mathematical Practices	Explanations and Examples
HS.G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Connections: *ETHS-S1C2-01*; *11-12.RST.4*	G	M III	*HS.MP.7.* Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use geometric simulation software to explore the connection between circles and the Pythagorean Theorem. **Examples:** • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Write an equation for a circle given that the endpoints of the diameter are (-2, 7) and (4, -8). • Find the center and radius of the circle $4x^2 + 4y^2 - 4x + 2y – 1 = 0$.
HS.G-GPE.2. Derive the equation of a parabola given a focus and directrix. Connections: *ETHS-S1C2-01*; *11-12.RST.4*	A II	M III	*HS.MP.7.* Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use geometric simulation software to explore parabolas. **Examples:** • Write and graph an equation for a parabola with focus (2, 3) and directrix $y = 1$.
HS.G-GPE.3. Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Connections: *ETHS-S1C2-01*; *11-12.RST.4*	+	+	*HS.MP.7.* Look for and make use of structure. HS.MP.8. Look for and express regularity in repeated reasoning.	Students may use geometric simulation software to explore conic sections. **Example:** • Write an equation in standard form for an ellipse with foci at (0, 5) and (2, 0) and a center at the origin.

Geometry: Expressing Geometric Properties with Equations (G-GPE)

Use coordinates to prove simple geometric theorems algebraically

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-GPE.4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* Connections: *ETHS-S1C2-01*; *9-10.WHST.1a-1e*; *11-12.WHST.1a-1e*	G	M III	*HS.MP.3* Construct viable arguments and critique the reasoning of others. .	Students may use geometric simulation software to model figures and prove simple geometric theorems. **Example:** • Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram.
HS.G-GPE.5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). Connection: *9-10.WHST.1a-1e*	G	M III	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Lines can be horizontal, vertical, or neither. Students may use a variety of different methods to construct a parallel or perpendicular line to a given line and calculate the slopes to compare the relationships.

Geometry: Expressing Geometric Properties with Equations (G-GPE)

Use coordinates to prove simple geometric theorems algebraically *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. Connections: *ETHS-S1C2-01*; *9-10.RST.3*	G	M III	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use geometric simulation software to model figures or line segments. **Examples:** • Given A(3, 2) and B(6, 11), ○ Find the point that divides the line segment AB two-thirds of the way from A to B. The point two-thirds of the way from A to B has *x*-coordinate two-thirds of the way from 3 to 6 and *y* coordinate two-thirds of the way from 2 to 11. So, (5, 8) is the point that is two-thirds from point A to point B. ○ Find the midpoint of line segment AB.
HS.G-GPE.7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. Connections: *ETHS-S1C2-01*; *9-10.RST.3*; *11-12.RST.3*	G ★	M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Students may use geometric simulation software to model figures.

Geometry: Geometric Measurement and Dimension (G-GMD)

Explain volume formulas and use them to solve problems

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-GMD.1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.* Connections: *9-10.RST.4; 9-10.WHST.1c; 9-10.WHST.1e; 11-12.RST.4; 11-12.WHST.1c; 11-12.WHST.1e*	G	M II	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.
HS.G-GMD.2. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. Connections: *9-10.RST.4*; *9-10.WHST.1c*; *9-10.WHST.1e*; *11-12.RST.4*; *11-12.WHST.1c*; *11-12.WHST.1e*	+	+	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. HS.MP.4. Model with mathematics. HS.MP.5. Use appropriate tools strategically.	Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.
HS.G-GMD.3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Connection: *9-10.RST.4*	G ★	M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively.	Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.

Geometry: Geometric Measurement and Dimension (G-GMD)

Visualize relationships between two-dimensional and three dimensional objects

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-GMD.4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. Connection: *ETHS-S1C2-01*	G	M III	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use geometric simulation software to model figures and create cross sectional views. **Example:** • Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.

Geometry: Geometric Measurement and Dimension ★ (G-MG)

Apply geometric concepts in modeling situations

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.G-MG.1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Connections: *ETHS-S1C2-01*; *9-10.WHST.2c*	G ★	M III ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use simulation software and modeling software to explore which model best describes a set of data or situation.
HS.G-MG.2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). Connection: *ETHS-S1C2-01*	G ★	M III ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use simulation software and modeling software to explore which model best describes a set of data or situation.
HS.G-MG.3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). Connection: *ETHS-S1C2-01*	G ★	M III ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use simulation software and modeling software to explore which model best describes a set of data or situation.

High School: Statistics and Probability Overview

Interpreting Categorical and Quantitative Data (S-ID)

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions (S-IC)

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability (S-CP)

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions (S-MD)

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Mathematics Standards – Mathematical Practices – Explanations and Examples

Statistics and Probability ★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling

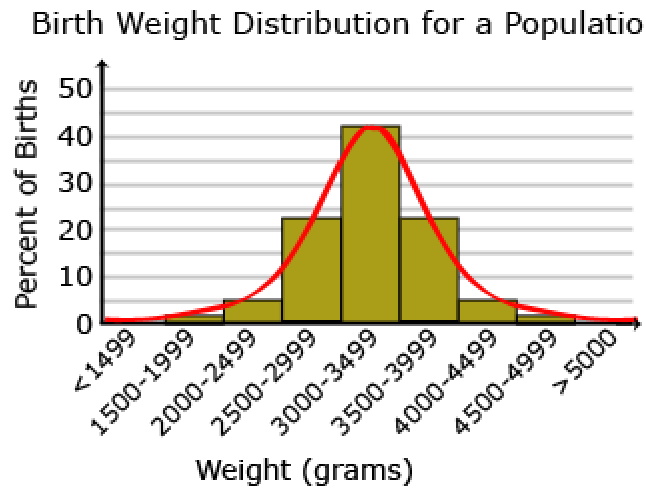
Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability: Interpreting Categorical and Quantitative Data★ (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots). Connections: *SCHS-S1C1-04*; *SCHS-S1C2-03*; *SCHS-S1C2-05*; *SCHS-S1C4-02*; *SCHS-S2C1-04*; *ETHS-S6C2-03*; *SSHS-S1C1-04*; *9-10.RST.7*	A I ★	M I ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	
HS.S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Connections: *SCHS-S1C3-06*; *ETHS-S6C2-03;SSHS-S1C1-01*	A I ★	M I ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics.HS.MP.5. Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets. **Examples:** • The two data sets below depict the housing prices sold in the King River area and Toby Ranch areas of Pinal County, Arizona. Based on the prices below which price range can be expected for a home purchased in Toby Ranch? In the King River area? In Pinal County? ○ King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000} ○ Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000} • Given a set of test scores: 99, 96, 94, 93, 90, 88, 86, 77, 70, 68, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher?

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID) **Summarize, represent, and interpret data on a single count or measurement variable** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Connections: *SSHS-S1C1-01*; *ETHS-S6C2-03*;*9-10.WHST.1a*	A I ★	M I ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use spreadsheets, graphing calculators and statistical software to statistically identify outliers and analyze data sets with and without outliers as appropriate.

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID) **Summarize, represent, and interpret data on a single count or measurement variable** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.7*; *11-12.RST.8*;*11-12.WRT.1b*	A II ★	M III ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use spreadsheets, graphing calculators, statistical software and tables to analyze the fit between a data set and normal distributions and estimate areas under the curve. **Examples:** • The bar graph below gives the birth weight of a population of 100 chimpanzees. The line shows how the weights are normally distributed about the mean, 3250 grams. Estimate the percent of baby chimps weighing 3000-3999 grams.  • Determine which situation(s) is best modeled by a normal distribution. Explain your reasoning. ○ Annual income of a household in the U.S. ○ Weight of babies born in one year in the U.S.

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*
HS.S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.9*; *11-12.WHST.1a-1b*; *11-12.WHST.1e*	A I ★	M I ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.8.* Look for and express regularity in repeated reasoning.

Explanations and Examples

Students may use spreadsheets, graphing calculators, and statistical software to create frequency tables and determine associations or trends in the data.

Examples:

Two-way Frequency Table

A two-way frequency table is shown below displaying the relationship between age and baldness. We took a sample of 100 male subjects, and determined who is or is not bald. We also recorded the age of the male subjects by categories.

Two-way Frequency Table			
Bald	Age		Total
	Younger than 45	45 or older	
No	35	11	46
Yes	24	30	54
Total	59	41	100

The *total* row and *total* column entries in the table above report the marginal frequencies, while entries in the body of the table are the joint frequencies.

Two-way Relative Frequency Table

The relative frequencies in the body of the table are called conditional relative frequencies.

Two-way Relative Frequency Table			
Bald	Age		Total
	Younger than 45	45 or older	
No	0.35	0.11	0.46
Yes	0.24	0.30	0.54
Total	0.59	0.41	1.00

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)

Summarize, represent, and interpret data on a single count or measurement variable

Standards Students are expected to:	TRAD	INT	Mathematical Practices	Explanations and Examples
HS.S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Connections: *SCHS-S1C2-05*; *SCHS-S1C3-01*; *ETHS-S1C2-01*; *ETHS-S1C3-01*; *ETHS-S6C2-03*	A I ★	M I M II M III ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	The residual in a regression model is the difference between the observed and the predicted y for some x(y the dependent variable and $x$the independent variable). So if we have a model $y = ax + b$, and a data point (x_i, y_i) the residual is for this point is: $r_i = y_i - (ax_i + b)$. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals. **Example:** • Measure the wrist and neck size of each person in your class and make a scatterplot. Find the least squares regression line. Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.
a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models.* Connection: *11-12.RST.7*	A I A II ★	M I M II M III ★		
b. Informally assess the fit of a function by plotting and analyzing residuals. Connections: *11-12.RST.7*; *11-12.WHST.1b-1c*	A I ★	M II M III ★		
c. Fit a linear function for a scatter plot that suggests a linear association. Connection: *11-12.RST.7*	A I ★	M I ★		

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)

Interpret linear models

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Connections: *SCHS-S5C2-01*; *ETHS-S1C2-01*;*ETHS-S6C2-03*; *9-10.RST.4*; *9-10.RST.7*; *9-10.WHST.2f*	A I ★	M I ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision.	Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models. **Example:** • Lisa lights a candle and records its height in inches every hour. The results recorded as (time, height) are (0, 20), (1, 18.3), (2, 16.6), (3, 14.9), (4, 13.2), (5, 11.5), (7, 8.1), (9, 4.7), and (10, 3). Express the candle's height (h) as a function of time (t) and state the meaning of the slope and the intercept in terms of the burning candle. Solution: $h = -1.7t + 20$ Slope: The candle's height decreases by 1.7 inches for each hour it is burning. Intercept: Before the candle begins to burn, its height is 20 inches.
HS.S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.5*; *11-12.WHST.2e*	A I ★	M I ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients. **Example:** • Collect height, shoe-size, and wrist circumference data for each student. Determine the best way to display the data. Answer the following questions: Is there a correlation between any two of the three indicators? Is there a correlation between all three indicators? What patterns and trends are apparent in the data? What inferences can be made from the data?

Statistics and Probability: Interpreting Categorical and Quantitative Data ★ (S-ID)

Interpret linear models *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-ID.9. Distinguish between correlation and causation. Connection: *9-10.RST.9*	A I ★	M I ★	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision.	Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment. **Example:** Diane did a study for a health class about the effects of a student's end-of-year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students' math scores and height. She concluded that "doing well on your end-of-course math tests makes you tall." Is this conclusion justified? Explain any flaws in Diane's reasoning.

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)

Understand and evaluate random processes underlying statistical experiments

Standards *Students are expected to:*	*TRAD*	INT	*Mathematical Practices*	*Explanations and Examples*
HS.S-IC.1. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.	A II ★	M III ★	*HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision.	

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)

Understand and evaluate random processes underlying statistical experiments *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin will fall heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?* Connections: *ETHS-S6C2-03*; *9-10.WHST.2d*; *9-10.WHST.2f*	A II ★	M III ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3*. Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7*. Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Possible data-generating processes include (but are not limited to): flipping coins, spinning spinners, rolling a number cube, and simulations using the random number generators. Students may use graphing calculators, spreadsheet programs, or applets to conduct simulations and quickly perform large numbers of trials. The law of large numbers states that as the sample size increases, the experimental probability will approach the theoretical probability. Comparison of data from repetitions of the same experiment is part of the model building verification process. **Example:** • Have multiple groups flip coins. One group flips a coin 5 times, one group flips a coin 20 times, and one group flips a coin 100 times. Which group's results will most likely approach the theoretical probability?

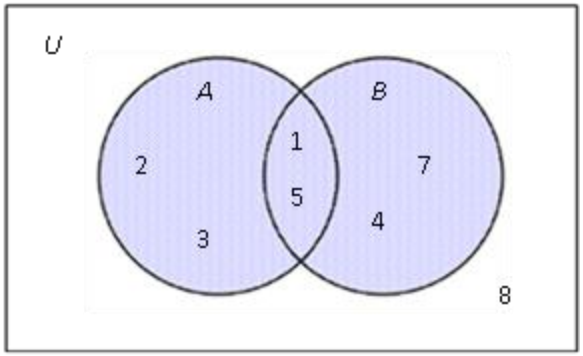
Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC)

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Connections: *11-12.RST.9*; *11-12.WHST.2b*	A II ★	M III ★	*HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision.	Students should be able to explain techniques/applications for randomly selecting study subjects from a population and how those techniques/applications differ from those used to randomly assign existing subjects to control groups or experimental groups in a statistical experiment. In statistics, an observational study draws inferences about the possible effect of a treatment on subjects, where the assignment of subjects into a treated group versus a control group is outside the control of the investigator (for example, observing data on academic achievement and socio-economic status to see if there is a relationship between them). This is in contrast to controlled experiments, such as randomized controlled trials, where each subject is randomly assigned to a treated group or a control group before the start of the treatment.
HS.S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Connections: *ETHS-S6C2-03*; *11-12.RST.9*; *11-12.WHST.1e*	A II ★	M III ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically.	Students may use computer generated simulation models based upon sample surveys results to estimate population statistics and margins of error.
HS.S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Connections:*ETHS-S6C2-03*; *11-12.RST.4*; *11-12.RST.5*; *11-12.WHST.1e*	A II ★	M III ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use computer generated simulation models to decide how likely it is that observed differences in a randomized experiment are due to chance. Treatment is a term used in the context of an experimental design to refer to any prescribed combination of values of explanatory variables. For example, one wants to determine the effectiveness of weed killer. Two equal parcels of land in a neighborhood are treated; one with a placebo and one with weed killer to determine whether there is a significant difference in effectiveness in eliminating weeds.

Statistics and Probability: Making Inferences and Justifying Conclusions ★ (S-IC) **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*

HS.S-IC.6. Evaluate reports based on data.
 	
Connections: *11-12.RST.4*; *11-12.RST.5*;*11-12.WHST.1b*; *11-12.WHST.1e*
	A II
	★
		M III
		★
			HS.MP.1. Make sense of problems and persevere in solving them.
			
			HS.MP.2. Reason abstractly and quantitatively.
			
			HS.MP.3. Construct viable arguments and critique the reasoning of others.
			
			HS.MP.4. Model with mathematics.
			
			HS.MP.5. Use appropriate tools strategically.
			
			HS.MP.6. Attend to precision.
			
			HS.MP.7. Look for and make use of structure.
			
			HS.MP.8. Look for and express regularity in repeated reasoning.
				Explanations can include but are not limited to sample size, biased survey sample, interval scale, unlabeled scale, uneven scale, and outliers that distort the line-of-best-fit. In a pictogram the symbol scale used can also be a source of distortion.
				
				As a strategy, collect reports published in the media and ask students to consider the source of the data, the design of the study, and the way the data are analyzed and displayed.
				
				Example:
				
				• A reporter used the two data sets below to calculate the mean housing price in Arizona as $629,000. Why is this calculation not representative of the typical housing price in Arizona?
				  ○ King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}
				  ○ Toby Ranch homes {5million, 154000, 250000, 250000, 200000, 160000, 190000}

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP) **Understand independence and conditional probability and use them to interpret data**				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). Connection: *11-12.WHST.2e*	A II ★	M II ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Intersection: The **intersection** of two sets *A* and *B* is the set of elements that are common to both set *A* **and** set *B*. It is denoted by $A \cap B$ and is read '*A* intersection *B*'. • $A \cap B$ in the diagram is {1, 5} • this means: BOTH/AND  Union: The **union** of two sets *A* and *B* is the set of elements, which are in *A* **or** in *B* **or** in both. It is denoted by $A \cup B$ and is read '*A* union *B*'. • $A \cup B$ in the diagram is {1, 2, 3, 4, 5, 7} • this means: EITHER/OR/ANY • *could* be both Complement: The **complement of the set $A \cup B$** is the set of elements that are members of the universal set U but are not in $A \cup B$. It is denoted by $(A \cup B)'$ • $(A \cup B)'$ in the diagram is {8}

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP) **Understand independence and conditional probability and use them to interpret data** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. Connection: *11-12.WHST.1e*	A II ★	M II ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	
HS.S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. Connections: *11-12.RST.5*; *11-12.WHST.1e*	A II ★	M II ★	*HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP)

Understand independence and conditional probability and use them to interpret data *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* Connections: *ETHS-S6C2-03*; *11-12.RST.4*; *11-12.RST.9*; *11-12.WHST.1e*	A II ★	M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use spreadsheets, graphing calculators, and simulations to create frequency tables and conduct analyses to determine if events are independent or determine approximate conditional probabilities.

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP)

Understand independence and conditional probability and use them to interpret data *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* Connections: *11-12.RST.4*; *11-12.RST.5*;*11-12.WHST.1e*	A II ★	M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.6.* Attend to precision. *HS.MP.8.* Look for and express regularity in repeated reasoning.	**Examples:** • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced? Are these events independent or dependent? • At Johnson Middle School, the probability that a student takes computer science and French is 0.062. The probability that a student takes computer science is 0.43. What is the probability that a student takes French given that the student is taking computer science?

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.9*; *11-12.WHST.1b*;*11-12.WHST.1e*	A II ★	M II ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.

Statistics and Probability: Conditional Probability and the Rules of Probability ★ (S-CP)

Use the rules of probability to compute probabilities of compound events in a uniform probability model *continued*

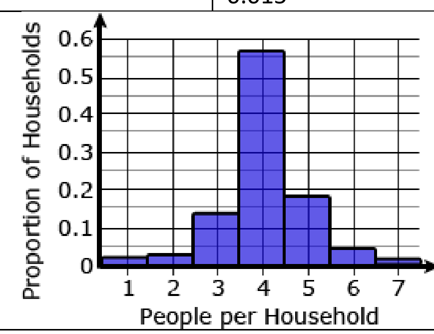
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-CP.7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*; *11-12.RST.9*	A II ★	M II ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes. **Example:** • In a math class of 32 students, 18 are boys and 14 are girls. On a unit test, 5 boys and 7 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?
HS.S-CP.8. Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.9*	+ ★	+ ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students could use graphing calculators, simulations, or applets to model probability experiments and interpret the outcomes.
HS.S-CP.9. Use permutations and combinations to compute probabilities of compound events and solve problems. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*; *11-12.RST.9*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use calculators or computers to determine sample spaces and probabilities. **Example:** • You and two friends go to the grocery store and each buys a soda. If there are five different kinds of soda, and each friend is equally likely to buy each variety, what is the probability that no one buys the same kind?

Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Calculate expected values and use them to solve problems

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. Connections: *ETHS-S6C2-03*; *11-12.RST.5*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use spreadsheets, graphing calculators and statistical software to represent data in multiple forms. **Example:** • Suppose you are working for a contractor who is designing new homes. She wants to ensure that the home models match the demographics for the area. She asks you to research the size of households in the region in order to better inform the floor plans of the home. Solution: • A possible solution could be the result of research organized in a variety of forms. In this case, the results of the research are shown in a table and graph. The student has defined their variable as x as the number of people per household.

People per Household	Proportion of Households
1	0.026
2	0.031
3	0.132
4	0.567
5	0.181
6	0.048
7	0.015



Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Calculate expected values and use them to solve problems *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*; *11-12.RST.3*; *11-12.RST.4*; *11-12.RST.9*	+ ★	+ ★	*HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure.	Students may use spreadsheets or graphing calculators to complete calculations or create probability models. The expected value of an uncertain event is the sum of the possible points earned multiplied by each point's chance of occurring. **Example:** • In a game, you roll a six sided number cube numbered with 1, 2, 3, 4, 5 and 6. You earn 3 points if a 6 comes up, 6 points if a 2, 4 or 5 come up and nothing otherwise. Since there is a 1/6 chance of each number coming up, the outcomes, probabilities and payoffs look like this: Outcome / Probability / Points: 1 / 1/6 / 0 points 2 / 1/6 / 6 points 3 / 1/6 / 0 points 4 / 1/6 / 6 points 5 / 1/6 / 6 points 6 / 1/6 / 3 points The expected value is sum of the products of the probability and points earned for each outcome (the entries in the last two columns multiplied together): $\left(\frac{1}{6}\right)\bullet 0+\left(\frac{1}{6}\right)\bullet 6+\left(\frac{1}{6}\right)\bullet 0+\left(\frac{1}{6}\right)\bullet 6+\left(\frac{1}{6}\right)\bullet 6+\left(\frac{1}{6}\right)\bullet 3=3.50\text{ points}$

Statistics and Probability: Using Probability to Make Decisions ★ (S-MD) **Calculate expected values and use them to solve problems** *continued*				
Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*;*11-12.RST.3*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.

Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Calculate expected values and use them to solve problems *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.

Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Use probability to evaluate outcomes of decisions

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. Connections: *SSHS-S5C2-03*, *SSHS-S5C5-03*, *SSHS-S5C5-05*; *ETHS-S1C2-01 ETHS-S6C2-03*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Different types of insurance to be discussed include but are not limited to: health, automobile, property, rental, and life insurance. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions
a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* Connections: 11-12.RST.3; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★		
b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* Connections: *11-12.RST.3*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★		

Statistics and Probability: Using Probability to Make Decisions ★ (S-MD)

Use probability to evaluate outcomes of decisions *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
HS.S-MD.6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*; *11-12.RST.3*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.
HS.S-MD.7. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). Connections: *ETHS-S1C2-01*; *ETHS-S6C2-03*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4*. Model with mathematics. *HS.MP.5*. Use appropriate tools strategically. *HS.MP.7.* Look for and make use of structure.	Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and interpret parameters in linear, quadratic or exponential functions.

High School: Contemporary Mathematics Overview (Arizona addition)

Discrete Mathematics (CM-DM)

- Understand and apply vertex-edge graph topics

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

High School: Contemporary Mathematics ★

Discrete mathematics is contemporary mathematics. This area of mathematics is very relevant in today's technologically advanced society. Discrete mathematics provides the underpinnings for many features of the Internet, from encryption of card numbers to decompression and compression of photographs, music, and video. It also informs the efficiency of our communication and transportation systems, such as determining the shortest path through a network or identifying the most cost effective design of airline or bus routes. The power of discrete mathematics is exemplified through the motivational impact on students. They are not only immersed in interesting mathematics but are actively engaged in the "doing" of mathematics. Mathematics is not a bystander sport.

Discrete mathematics topics, particularly vertex-edge graphs, afford students the opportunity to access problem solving in a meaningful context. Students strengthen their skills in problem solving, reasoning, conjecturing, communication, analysis, and proof. They apply the Standards for Mathematical Practice as they solve discrete mathematics problems. Discrete mathematics courses play an increasingly important role in the high school curriculum as possible pathways for those students who seek meaningful + courses that connect to technology and the needs of the 21st century learner.

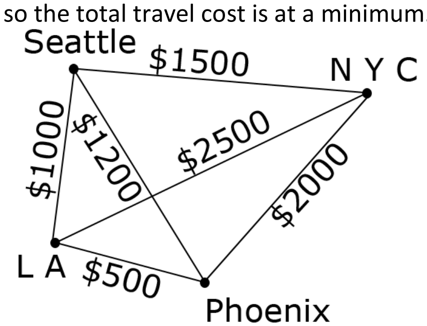
Graph theory is the formal study of vertex-edge graphs. Unlike graphs used in data analysis, vertex-edge graphs are used to visually represent problem situations. Vertex-edge graphs are used to model and solve problems related to paths, circuits, or the relationship among a set of objects.

Connections to Modeling

Mathematical modeling occurs when students follow a multistep process of solving problems and represent the key ideas through a visual representation. These visual representations allow students multiple entry points for solving a problem, ensuring material that is both engaging and accessible. Examples of real word situations that could be modeled using a vertex-edge graph are 1) planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player or 2) engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.

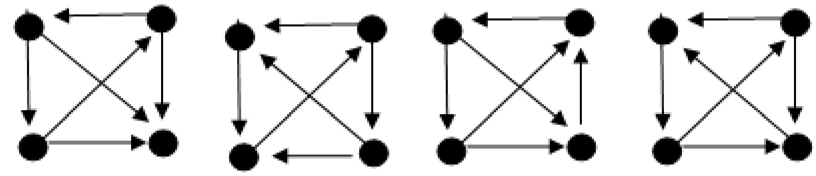
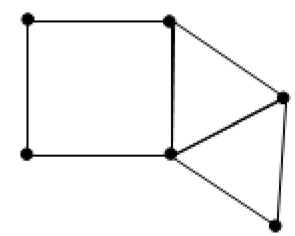
Contemporary Mathematics: Discrete Mathematics ★ (CM-DM)

Understand and apply vertex-edge graph topics

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
AZ.HS.CM-DM.1. Study the following topics related to vertex-edge graphs: Euler circuits, Hamilton circuits, the Travelling Salesperson Problem (TSP), minimum weight spanning trees, shortest paths, vertex coloring, and adjacency matrices. Connections: *ETHS-S6C2-03*; *11-12.RST.4*; *11-12.RST.5*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use graphing calculators or computer algebra systems to assist with computations. **Examples:** • A businesswoman in Phoenix is planning a trip to visit clients in Seattle, Los Angeles and New York City before returning to Phoenix. The figure below gives the cost in dollars of traveling from one city to another. Find the order in which these cities should be visited so the total travel cost is at a minimum.  Note that the businesswoman's trip is the same as a circuit that starts at vertex 1 (Phoenix), visits each other vertex exactly once, and returns to vertex 1. In other words, the circuit is a Hamiltonian circuit, and the businesswoman's task is to find the Hamiltonian circuit of least total weight (given the weighted graph) *Continued on next page*

Contemporary Mathematics: Discrete Mathematics ★ (CM-DM)

Understand and apply vertex-edge graph topics *continued*

Standards *Students are expected to:*	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
AZ.HS.CM-DM.1. *continued*				• Which directed graph below represents a tournament on four vertices, where all players but one are champions?  Graph 1 Graph 2 Graph 3 Graph 4 • Build a tournament on 5 vertices where all players but one are champions. • Juanita claims that the graph below has an Euler path but not an Euler circuit. Justify her claim. 

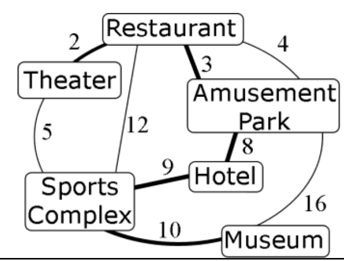
Contemporary Mathematics: Discrete Mathematics ★ (CM-DM)

Understand and apply vertex-edge graph topics *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
AZ.HS.CM-DM.2. Understand, analyze, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings. Connections: *ETHS-S6C2-03*; *11-12.RST.9*; *11-12.WHST.1b*; *11-12.WHST.1e*;	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	Students may use graphing calculators or computer algebra systems to assist with computations. **Examples:** • Find a minimal route that includes every street (e.g., for trash pick-up). • Find the shortest network connecting specified sites.

Contemporary Mathematics: Discrete Mathematics ★ (CM-DM)

Understand and apply vertex-edge graph topics *continued*

Standards Students are expected to:	*INT*	*TRAD*	*Mathematical Practices*	*Explanations and Examples*
AZ.HS.CM-DM.3. Devise, analyze, and apply algorithms for solving vertex-edge graph problems. Connections: *ETHS-S6C2-03*; *11-12.RST.3*; *11-12.RST.4*; *11-12.RST.9*; *11-12.WHST.1a*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning	In exploring minimum spanning tree situations students devise, analyze, and apply algorithms as they adopt strategies to confront the problem. Such strategies can lead to Kruskal's algorithm, Prim's algorithm, or the "nearest neighbor" algorithm. Students may use graphing calculators or computer algebra systems to assist with computations. **Example:** • Susan is a city planner in charge of the development of roads for a recreational area. The graph shows locations in the area, the possible roads that could be built between locations, and the cost in thousands of dollars to build each road. Find the smallest possible cost of building enough roads to connect the locations. **Algorithm to Find a Minimum Spanning Tree in a Connected Graph** Given a connected graph with weights on the edges: Step 1. List the edges of the graph by increasing weights. Step 2. Choose the edge with the smallest weight. Step 3. Continue to choose the next edge with the smallest weight as long as choosing that edge does not create a circuit. Step 4. Stop when the result is a spanning tree. The graph shown is the original graph and also shows the spanning tree (bolded edges) that would be produced by applying the algorithm. The smallest possible cost to build roads connecting all the sites would be to build a road between the theater and restaurant (2), between the restaurant and amusement park (3), between the amusement park and hotel (8), between the hotel and the sports complex (9), and between the sports complex and the museum (10). There is a minimum total cost of $32,000 to build the roads at the recreational area. 

Contemporary Mathematics: Discrete Mathematics ★ (CM-DM)

Understand and apply vertex-edge graph topics *continued*

Standards Students are expected to:	*TRAD*	*INT*	*Mathematical Practices*	*Explanations and Examples*
AZ.HS.CM-DM.4. Extend work with adjacency matrices for graphs, such as interpreting row sums and using the nth power of the adjacency matrix to count paths of length *n* in a graph. Connections: *ETHS-S6C2-03*; *11-12.RST.4*; *11-12.RST.5*; *11-12.RST.9*; *11-12.WHST.1a*; *11-12.WHST.1b*; *11-12.WHST.1e*	+ ★	+ ★	*HS.MP.1.* Make sense of problems and persevere in solving them. *HS.MP.2.* Reason abstractly and quantitatively. *HS.MP.3.* Construct viable arguments and critique the reasoning of others. *HS.MP.4.* Model with mathematics. *HS.MP.5.* Use appropriate tools strategically. *HS.MP.6.* Attend to precision. *HS.MP.7.* Look for and make use of structure. *HS.MP.8.* Look for and express regularity in repeated reasoning.	The adjacency matrix of a simple graph is a matrix with rows and columns labeled by graph vertices, with a 1 or a 0 in position ($\mathbf{v}_i$, $\mathbf{v}_j$) according to whether $\mathbf{v}_i$ and $\mathbf{v}_j$ are adjacent or not. A "1" indicates that there is a connection between the two vertices, and a "0" indicates that there is no connection. Students may use graphing calculators or computer algebra systems to assist with computations.

Standards for Mathematical Practice (MP)

Standards Students are expected to:	*Mathematical Practices* are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.	*Explanations and Examples*
HS.MP.1. Make sense of problems and persevere in solving them.		High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
HS.MP.2. Reason abstractly and quantitatively.		High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
HS.MP.3. Construct viable arguments and critique the reasoning of others.		High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains, to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Standards for Mathematical Practice (MP) *continued*		
Standards ***Students are expected to:***	***Mathematical Practices*** ***are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.***	***Explanations and Examples***
HS.MP.4. Model with mathematics.		High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
HS.MP.5. Use appropriate tools strategically.		High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
HS.MP.6. Attend to precision.		High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Standards for Mathematical Practice (MP) *continued*		
Standards ***Students are expected to:***	*Mathematical Practices* ***are listed throughout the grade level document in the 2nd column to reflect the need to connect the mathematical practices to mathematical content in instruction.***	*Explanations and Examples*
HS.MP.7. Look for and make use of structure.		By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
HS.MP.8. Look for and express regularity in repeated reasoning.		High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x2 + x + 1)$, and $(x – 1)(x3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

ASSESSMENT LIMITS FOR STANDARDS ASSESSED ON MORE THAN ONE END-OF-COURSE TEST: AI-G-AII PATHWAY

Table 1. This draft table shows assessment limits for standards assessed on more than one end-of-course test.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
Reason quantitatively and use units to solve problems	N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.	This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
Interpret the structure of expressions	A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	i) Tasks are limited to numerical expressions and polynomial expressions in one variable. ii) Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form (53+47)(53–47). See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$.	i) Tasks are limited to polynomial, rational, or exponential expressions. ii) Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (–1, 0). See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
Write expressions in equivalent forms to solve problems	A-SSE.3c	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.↔ (c) Use the properties of exponents to transform expressions for exponential	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent	i) Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
		functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with integer exponents.	form of the expression reveals something about the situation. ii) Tasks are limited to exponential expressions with rational or real exponents.
Understand the relationship between zeros and factors of polynomials	A-APR.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	i) Tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.	i) Tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 - 1)(x^2 + 1)$
Create equations that describe numbers or relationships	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	i) Tasks are limited to linear, quadratic, or exponential equations with integer exponents.	i) Tasks are limited to exponential equations with rational or real exponents and rational functions. ii) Tasks have a real-world context.
Understand solving equations as a process of reasoning and explain the reasoning	A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	i) Tasks are limited to quadratic equations.	i) Tasks are limited to simple rational or radical equations.
Solve equations and inequalities in one variable	A-REI.4b	Solve quadratic equations in one variable. b) Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	i) Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions. *Note, solving a quadratic equation by factoring relies on the connection between zeros and factors of polynomials (cluster A-APR.B). Cluster A-APR.B is formally assessed in A2.*	i) In the case of equations that have roots with nonzero imaginary parts, students write the solutions as a ± bi for real numbers a and b.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
Solve systems of equations	A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	i) Tasks have a real-world context. ii) Tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	i) Tasks are limited to 3x3 systems.
Represent and solve equations and inequalities graphically	A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x) =g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial functions.	i) Tasks may involve any of the function types mentioned in the standard.
Understand the concept of a function and use function notation	F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.*	i) This standard is part of the Major work in Algebra I and will be assessed accordingly.	i) This standard is Supporting work in Algebra II. This standard should support the Major work in F-BF.2 for coherence.
Interpret functions that arise in applications in terms of a context	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *Compare note (ii) with standard F-IF.7.* *The function types listed here are the*	i) Tasks have a real-world context ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *Compare note (ii) with standard F-IF.7.* *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.6 and F-IF.9.*

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
			same as those listed in the Algebra I column for standards F-IF.6 and F-IF.9.	
Interpret functions that arise in applications in terms of a context	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.9.*
Analyze functions using different representations	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	i) Tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Algebra I column for standards F-IF.4 and F-IF.6.*	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Algebra II column for standards F-IF.4 and F-IF.6.*
Build a function that models a relationship between two quantities	F-BF.1a	Write a function that describes a relationship between two quantities.★ a) Determine an explicit expression, a recursive process, or steps for calculation from a context.	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.	i) Tasks have a real-world context ii) Tasks may involve linear functions, quadratic functions, and exponential functions.
Build new functions from existing functions	F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with	i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative) is limited to linear and quadratic functions.	i) Tasks may involve polynomial, exponential, logarithmic, and trigonometric functions ii) Tasks may involve recognizing even and odd functions.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Algebra I Assessment Limits and Clarifications	Algebra II Assessment Limits and Clarifications
		cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*	ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. iii) Tasks do not involve recognizing even and odd functions. *The function types listed in note (ii) are the same as those listed in the Algebra I column for standards F-IF.4, F-IF.6, and F-IF.9.*	*The function types listed in note (i) are the same as those listed in the Algebra II column for standards F-IF.4, F-IF.6, and F-IF.9.*
Construct and compare linear, quadratic, and exponential models and solve problems	F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	i) Tasks are limited to constructing linear and exponential functions in simple context (not multi-step).	i) Tasks will include solving multi-step problems by constructing linear and exponential functions.
Interpret expressions for functions in terms of the situation they model	F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.	i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers.
Summarize, represent, and interpret data on two categorical and quantitative variables	S-ID.6a	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	i) Tasks have a real-world context. ii) Exponential functions are limited to those with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.

ASSESSMENT LIMITS FOR STANDARDS ASSESSED ON MORE THAN ONE END-OF-COURSE TEST: MATHEMATICS I - III PATHWAY

Table 2. This draft table shows assessment limits for standards assessed on more than one end-of-course test.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
Reason quantitatively and use units to solve problems	N-Q.2	Define appropriate quantities for the purpose of descriptive modeling.	This standard will be assessed in Math I by ensuring that some modeling tasks (involving Math I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving data, the student might autonomously decide that a measure of center is a key variable in a situation, and then choose to work with the mean.	This standard will be assessed in Math II by ensuring that some modeling tasks (involving Math II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving volume of a prism or pyramid, the student might autonomously decide that the area of the base is a key variable in a situation, and then choose to work with that dimension to solve the problem.	This standard will be assessed in Math III by ensuring that some modeling tasks (involving Math III content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., a quantity of interest is not selected for the student by the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
Interpret the structure of expressions	A-SSE.1b	Interpret expressions that represent a quantity in terms of its context. b) Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)n*	i) Tasks are limited to exponential expressions, including related numerical expressions.	i) Tasks are limited to quadratic expressions.	-

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		as the product of P and a factor not depending on P.			
Interpret the structure of expressions	A-SSE.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.	-	i) Tasks are limited to quadratic and exponential expressions, including related numerical expressions. ii) Examples: See an opportunity to rewrite $a^2 + 9a + 14$ as $(a+7)(a+2)$. Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53+47)(53-47)$.	i) Tasks are limited to polynomial and rational expressions. ii) Examples: see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center $(-1, 0)$. See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$.
Create equations that describe numbers or relationships	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	i) Tasks are limited to linear or exponential equations with integer exponents. ii) Tasks have a real-world context. iii) In the linear case, tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	i) Tasks are limited to quadratic and exponential equations. ii) Tasks have a real-world context. iii) In simpler cases (such as exponential equations with integer exponents), tasks have more of the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	i) Tasks are limited to simple rational or exponential equations ii) Tasks have a real-world context.
Create equations that describe numbers or	A-CED.2	Create equations in two or more variables to	i) Tasks are limited to linear equations ii) Tasks have a real-world	i) Tasks are limited to quadratic equations ii) Tasks have a real-world	i) Tasks are limited to simple polynomial, rational, or exponential equations

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
relationships		represent relationships between quantities; graph equations on coordinate axes with labels and scales	context. iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	context. iii) Tasks have the hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).	ii) Tasks have a real-world context.
Create equations that describe numbers or relationships	A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	i) Tasks are limited to linear equations ii) Tasks have a real-world context.	i) Tasks are limited to quadratic equations ii) Tasks have a real-world context.	-
Understand solving equations as a process of reasoning and explain the reasoning	A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a	-	i) Tasks are limited to quadratic equations.	i) Tasks are limited to simple rational or radical equations.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		solution method.			
Represent and solve equations and inequalities graphically	A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial.	-	i) Tasks may involve any of the function types mentioned in the standard.
Interpret functions that arise in applications in terms of the context	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. *The function types listed here are the same as those listed in the Math II column for standards*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in*

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		terms of the quantities, and sketch graphs showing key features given a verbal description of the of the relationship. *Key features include; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimum; symmetries; end behavior; and periodicity.* ⋆	functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards F-IF.6 and F-IF.9.*	*F-IF.6 and F-IF.9.*	*the Math III column for standards F-IF.6 and F-IF.9.*
Interpret functions that arise in applications in terms of the context	F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the*	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to quadratic functions.	-

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		positive integers would be an appropriate domain for the function.			
Interpret functions that arise in applications in terms of the context	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) or a specified interval. Estimate the rate of change from a graph.	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. *The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.9.*
Analyze functions using different representations	F-IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ a) Graph linear and quadratic functions and show intercepts, maxima, and	i) Tasks are limited to linear functions.	i) Tasks are limited to quadratic functions.	-

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		minima.			
Analyze functions using different representations	F-IF.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ e) graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	-	i) Tasks are limited to exponential functions.	i) Tasks are limited to logarithmic and trigonometric functions.
Analyze functions using different representations	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions.) *For example, given a graph of one quadratic function*	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards*	i) Tasks are limited to on quadratic and exponential functions. ii) Tasks do not have a real-world context. *The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.6.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.6.*

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		and an algebraic expression for another, say which has the larger maximum.	*F-IF.4 and F-IF.6.*		
Build a function that models a relationship between two quantities	F-BF.1a	Write a function that describes a relationship between two quantities. ★ a) Determine an explicit expression, a recursive process, or steps for a calculation from a context	i) Tasks have a real-world context. ii) Tasks are limited to linear functions and exponential functions with domains in the integers.	i) Tasks have a real-world context. ii) Tasks may involve linear functions, quadratic functions, and exponential functions.	
Build new functions from existing functions	F-BF.3	Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x+k)* for specific values of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions*	-	i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. ii) Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions.	i) Tasks are limited to exponential, polynomial,logarithmic, and trigonometric functions. ii) Tasks may involve recognizing even and odd functions. *The function types listed in note (i) are the same as those listed in the Math III column for standards F-IF.4, F-IF.6, and F-IF.9.*

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		from their graphs and algebraic expressions for them.		iii) Tasks do not involve recognizing even and odd functions. *The function types listed in note (ii) are the same as those listed in the Math I and Math II columns for standards F-IF.4, F-IF.6, and F-IF.9.*	
Represent and solve equations and inequalities graphically	A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations y=f(x) and y=g(x) intersect are the solutions of the equation f(x)=g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	i) Tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. ii) Finding the solutions approximately is limited to cases where f(x) and g(x) are polynomial.	-	i) Tasks may involve any of the function types mentioned in the standard.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
Interpret functions that arise in applications in terms of the context	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the of the relationship. *Key features include; intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimum; symmetries; end behavior; and periodicity.* ⋆	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards F-IF.6 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. *The function types listed here are the same as those listed in the Math II column for standards F-IF.6 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Math III column for standards F-IF.6 and F-IF.9.*
Interpret functions that arise in applications in terms of the context	F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step	i) Tasks have a real-world context. ii) Tasks are limited to quadratic functions.	-

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	functions and absolute value functions), and exponential functions with domains in the integers.		
Interpret functions that arise in applications in terms of the context	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) or a specified interval. Estimate the rate of change from a graph.	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks are limited to quadratic and exponential functions. *The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.9.*	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions. *The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.9.*
Analyze functions using different representations	F-IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using	i) Tasks are limited to linear functions.	i) Tasks are limited to quadratic functions.	-

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		technology for more complicated cases. ★ a) Graph linear and quadratic functions and show intercepts, maxima, and minima.			
Analyze functions using different representations	F-IF.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ★ e) graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	-	i) Tasks are limited to exponential functions.	i) Tasks are limited to logarithmic and trigonometric functions.
Analyze functions using different representations	F-IF.9	Compare properties of two functions each represented in a different way (algebraically,	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, square root functions, cube root functions, piecewise-defined functions (including step	i) Tasks are limited to on quadratic and exponential functions. ii) Tasks do not have a real-world context.	i) Tasks have a real-world context. ii) Tasks may involve polynomial, logarithmic, and trigonometric functions.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		graphically, numerically in tables, or by verbal descriptions.) *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	functions and absolute value functions), and exponential functions with domains in the integers. *The function types listed here are the same as those listed in the Math I column for standards F-IF.4 and F-IF.6.*	*The function types listed here are the same as those listed in the Math II column for standards F-IF.4 and F-IF.6.*	*The function types listed here are the same as those listed in the Math III column for standards F-IF.4 and F-IF.6.*
Build a function that models a relationship between two quantities	F-BF.1a	Write a function that describes a relationship between two quantities. ⋆ a) Determine an explicit expression, a recursive process, or steps for a calculation from a context	i) Tasks have a real-world context. ii) Tasks are limited to linear functions and exponential functions with domains in the integers.	i) Tasks have a real-world context. ii) Tasks may involve linear functions, quadratic functions, and exponential functions.	
Build new functions from existing functions	F-BF.3	Identify the effect on the graph of replacing *f(x)* by *f(x) + k, k f(x), f(kx),* and *f(x+k)* for specific values of k given the graphs. Experiment with cases and	-	i) Identifying the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. ii) Experimenting with cases and illustrating an explanation of the effects on the graph using	i) Tasks are limited to exponential, polynomial,logarithmic, and trigonometric functions. ii) Tasks may involve recognizing even and odd functions. *The function types listed in note (i) are the same as those listed in the Math III column for*

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*		technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions. iii) Tasks do not involve recognizing even and odd functions. *The function types listed in note (ii) are the same as those listed in the Math I and Math II columns for standards F-IF.4, F-IF.6, and F-IF.9.*	*standards F-IF.4, F-IF.6, and F-IF.9.*
Summarize, represent, and interpret data on two categorical and quantitative variables	S-ID.6a	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a) Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context.	i) Tasks have real-world context. ii) Tasks are limited to linear functions and exponential functions with domains in the integers.	i) Tasks have real-world context. ii) Tasks are limited to quadratic functions.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.

ACCS-M Cluster	ACCS-M Key	ACCS-M Standard	Mathematics I Assessment Limits and Clarifications	Mathematics II Assessment Limits and Clarifications	Mathematics III Assessment Limits and Clarifications
		Emphasize linear, quadratic, and exponential models.			
Summarize, represent, and interpret data on two categorical and quantitative variables	S-ID.6b	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. b) Informally assess the fit of a function by plotting and analyzing residuals.	-	i) Tasks have a real-world context. ii) Tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.	i) Tasks have a real-world context. ii) Tasks are limited to exponential functions with domains not in the integers and trigonometric functions.