



## ACT Course Standards Geometry

A set of empirically derived course standards is the heart of each QualityCore<sup>®</sup> mathematics course. The ACT Course Standards represent a solid evidence-based foundation in mathematics. They were developed from an intensive study of high-performing high schools with significant minority and low-income enrollments that produced many graduates who met or exceeded ACT College Readiness Benchmark Scores (See <http://www.act.org/path/policy/reports/success.html>).

This document contains a list of ACT Course Standards for a rigorous Geometry course—what students should know and be able to do in the course—and a worksheet teachers can use to compare their course content to these standards. The ACT standards encompass the following overarching themes and/or foundational concepts:

- A. Prerequisites
- B. Exploring the Skills and Strategies Underlying Mathematics
- C. Using Logic and Proof to Reason Mathematically
- D. Identifying, Classifying, and Applying the Properties of Geometric Figures in Space
- E. Comparing Congruent and Similar Geometric Figures
- F. Using Length, Area, Perimeter, and Volume to Find Quantities and Solve Problems
- G. Relating Geometric Ideas to the Coordinate Plane
- H. Investigating and Applying Basic Ideas of Trigonometry

### ACT Course Standards—Geometry

A. Prerequisites	
1. Skills Acquired by Students in a Previous Course and Refined in This Course	
a.	Apply algebraic properties (e.g., commutative, associative, distributive, identity, inverse, substitution) to simplify algebraic expressions
b.	Solve single-step and multistep equations and inequalities in one variable
c.	Write linear equations in standard form and slope-intercept form when given two points, a point and the slope, or the graph of the equation
d.	Recognize the concept of slope as a rate of change and determine the slope when given the equation of a line in standard form or slope-intercept form, the graph of a line, two points, or a verbal description
e.	Graph a linear equation using a table of values, $x$ - and $y$ -intercepts, or slope-intercept form
f.	Find the probability of a simple event

## B. Exploring the Skills and Strategies Underlying Mathematics

### 1. Mathematical Processes Learned in the Context of Increasingly Complex Mathematical and Real-World Problems

(Note: These mathematical processes are the same for Algebra I, Geometry, Algebra II, and Precalculus.)

- a. Apply problem-solving skills (e.g., identifying irrelevant or missing information, making conjectures, extracting mathematical meaning, recognizing and performing multiple steps when needed, verifying results in the context of the problem) to the solution of real-world problems
- b. Use a variety of strategies to set up and solve increasingly complex problems
- c. Represent data, real-world situations, and solutions in increasingly complex contexts (e.g., expressions, formulas, tables, charts, graphs, relations, functions) and understand the relationships
- d. Use the language of mathematics to communicate increasingly complex ideas orally and in writing, using symbols and notations correctly
- e. Make appropriate use of estimation and mental mathematics in computations and to determine the reasonableness of solutions to increasingly complex problems
- f. Make mathematical connections among concepts, across disciplines, and in everyday experiences
- g. Demonstrate the appropriate role of technology (e.g., calculators, software programs) in mathematics (e.g., organize data, develop concepts, explore relationships, decrease time spent on computations after a skill has been established)
- h. Apply previously learned algebraic concepts in geometric contexts

## C. Using Logic and Proof to Reason Mathematically

### 1. Logic and Proof

- a. Use definitions, basic postulates, and theorems about points, segments, lines, angles, and planes to write proofs and to solve problems
- b. Use inductive reasoning to make conjectures and deductive reasoning to arrive at valid conclusions
- c. Identify and write conditional and biconditional statements along with the converse, inverse, and contrapositive of a conditional statement; use these statements to form conclusions
- d. Use various methods to prove that two lines are parallel or perpendicular (e.g., using coordinates, angle measures)
- e. Read and write different types and formats of proofs including two-column, flowchart, paragraph, and indirect proofs
- f. Prove that two triangles are congruent by applying the SSS, SAS, ASA, AAS, and HL congruence statements
- g. Use the principle that corresponding parts of congruent triangles are congruent to solve problems
- h. Use several methods, including AA, SAS, and SSS, to prove that two triangles are similar, corresponding sides are proportional, and corresponding angles are congruent
- i. Use properties of special quadrilaterals in a proof

## D. Identifying, Classifying, and Applying the Properties of Geometric Figures in Space

### 1. Points, Lines, Planes, and Space

- a. Identify and model plane figures, including collinear and noncollinear points, lines, segments, rays, and angles using appropriate mathematical symbols
- b. Identify vertical, adjacent, complementary, and supplementary angle pairs and use them to solve problems (e.g., solve equations, use in proofs)
- c. Identify corresponding, same-side interior, same-side exterior, alternate interior, and alternate exterior angle pairs formed by a pair of parallel lines and a transversal and use these special angle pairs to solve problems (e.g., solve equations, use in proofs)

d. Use construction techniques, including straightedge and compass, to bisect and trisect segments and to create parallel and perpendicular lines, perpendicular bisectors, and angle bisectors
e. Locate, describe, and draw a locus in a plane or space
f. Apply properties and theorems of parallel and perpendicular lines to solve problems
<b>2. Polygons</b>
a. Identify and classify triangles by their sides and angles
b. Identify medians, altitudes, perpendicular bisectors, and angle bisectors of triangles and use their properties to solve problems (e.g., find points of concurrency, segment lengths, or angle measures)
c. Apply the Triangle Inequality Theorem to determine if a triangle exists and the order of sides and angles
d. Solve problems involving the relationships formed when the altitude to the hypotenuse of a right triangle is drawn
e. Apply the Pythagorean Theorem and its converse to triangles to solve mathematical and real-world problems (e.g., shadows and poles, ladders)
f. Identify and use Pythagorean triples in right triangles to find lengths of the unknown side
g. Identify and classify quadrilaterals, including parallelograms, rectangles, rhombi, squares, kites, trapezoids, and isosceles trapezoids, using their properties
h. Identify and classify regular and nonregular polygons (e.g., pentagons, hexagons, heptagons, octagons, nonagons, decagons, dodecagons) based on the number of sides, the angle measures, and the side lengths
i. Apply the Angle Sum Theorem for triangles and polygons to find interior and exterior angle measures given the number of sides, to find the number of sides given angle measures, and to solve real-world problems
j. Apply the Isosceles Triangle Theorem and its converse to triangles to solve mathematical and real-world problems
<b>3. Circles</b>
a. Identify and define line segments associated with circles (e.g., radii, diameters, chords, secants, tangents)
b. Determine the measure of central and inscribed angles and their intercepted arcs
c. Find segment lengths, angle measures, and intercepted arc measures formed by chords, secants, and tangents intersecting inside and outside circles
d. Solve problems using inscribed and circumscribed polygons
<b>4. Solids</b>
a. Identify and classify prisms, pyramids, cylinders, cones, and spheres and use their properties to solve problems
b. Describe and draw cross sections of prisms, cylinders, pyramids, and cones
<b>E. Comparing Congruent and Similar Geometric Figures</b>
<b>1. Similarity and Congruence</b>
a. Determine points or lines of symmetry and apply the properties of symmetry to figures
b. Identify congruent figures and their corresponding parts
c. Identify similar figures and use ratios and proportions to solve mathematical and real-world problems (e.g., finding the height of a tree using the shadow of the tree and the height and shadow of a person)
d. Use the definition of similarity to establish the congruence of angles, proportionality of sides, and scale factor of two similar polygons
e. Identify and draw images of transformations and use their properties to solve problems
f. Apply relationships between perimeters of similar figures, areas of similar figures, and volumes of similar figures, in terms of scale factor, to solve mathematical and real-world problems
g. Determine the geometric mean between two numbers and use it to solve problems (e.g., find the lengths of segments in right triangles)
h. Identify and give properties of congruent or similar solids

## F. Using Length, Area, Perimeter, and Volume to Find Quantities and Solve Problems

### 1. Area and Perimeter

- a. Find the perimeter and area of common plane figures, including triangles, quadrilaterals, regular polygons, and irregular figures, from given information using appropriate units of measurement
- b. Manipulate perimeter and area formulas to solve problems (e.g., finding missing lengths)
- c. Use area to solve problems involving geometric probability
- d. Find arc lengths and circumferences of circles from given information (e.g., radius, diameter, coordinates)
- e. Find the area of a circle and the area of a sector of a circle from given information (e.g., radius, diameter, coordinates)

### 2. Lateral Area, Surface Area, and Volume

- a. Find the lateral area, surface area, and volume of prisms, cylinders, cones, and pyramids in mathematical and real-world settings
- b. Use cross sections of prisms, cylinders, pyramids, and cones to solve volume problems
- c. Find the surface area and volume of a sphere in mathematical and real-world settings

## G. Relating Geometric Ideas to the Coordinate Plane

### 1. Coordinate Geometry

- a. Use slope to distinguish between and write equations for parallel and perpendicular lines
- b. Apply the midpoint and distance formulas to points and segments to find midpoints, distances, and missing information
- c. Use coordinate geometry to solve problems about geometric figures (e.g., segments, triangles, quadrilaterals)
- d. Write equations for circles in standard form and solve problems using equations and graphs
- e. Determine the effect of reflections, rotations, translations, and dilations and their compositions on the coordinate plane

## H. Investigating and Applying Basic Ideas of Trigonometry

### 1. Introduction to Trigonometry

- a. Apply properties of  $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles to determine lengths of sides of triangles
- b. Find the sine, cosine, and tangent ratios of acute angles given the side lengths of right triangles
- c. Use trigonometric ratios to find the sides or angles of right triangles and to solve real-world problems (e.g., use angles of elevation and depression to find missing measures)

## ACT Course Standards Worksheet—Geometry

This worksheet gives teachers an opportunity to compare their course content to ACT's QualityCore® program. Completing the worksheet also allows teachers who teach the same course to ensure their courses have similar outcomes.

### Gap Analysis 1—Individual Teacher Review

This analysis allows individual teachers to identify “gaps” between ACT Course Standards and their course content. They should review the ACT standards on the following worksheet, then determine whether the ACT standard **is** or **is not** included in the course as it is currently taught. “Included” means the standard is taught and students are expected to demonstrate proficiency by the end of the course. “Not Included” means the standard is not taught in the course, is taught in another course, or is already mastered. In the “Gap 1” column on the worksheet, place an “I” for “Included” or an “NI” for “Not Included.” Analyze any gaps between the current course standards and the ACT Course Standards. Identify reasons the standards receiving a “Not Included” designation are not included in the course.

### Gap Analysis 2—Group Consensus

This analysis allows groups of teachers who teach the same course and who have completed Gap Analysis 1 individually to identify differences in how they evaluated the gaps between ACT Course Standards and current course standards. In the “Gap 2” column of the worksheet, place an “X” where members of the group differed in their assessment of whether a particular ACT standard is included in the course as it is currently taught.

The following questions can guide discussion of the gaps:

#### Overarching Questions

1. What should students know and be able to do before going to the next course?
2. Do all teachers teaching this course have a shared understanding of the intent or meaning of each course standard and topic area?

#### Gap Analysis 1 Questions

1. Which ACT Course Standards were identified as not included in the course?
2. What is the level of agreement among the group of teachers about the skills and knowledge that is or is not taught in the course?
3. Are there sound pedagogical reasons for not including specific ACT standards in the course?
4. What implications will any decisions have on students' future learning and academic achievement?

#### Gap Analysis 2 Questions

1. Which of the ACT Course Standards elicited differences of opinion?
2. What are the possible reasons for different opinions about the standards that are or are not included in the course?
3. Are there sound pedagogical reasons for including or not including these disputed standards in the course?
4. What implications will any decisions have on students' future learning and academic achievement?

Finally, document the necessary steps to address the outcomes of the discussion. Be sure to note whether course standards will be added, deleted, or modified; identify who will be responsible for communicating any changes to other teachers; and note any other decisions. Document responsibilities and establish a timetable for continuing the discussion and implementing the decisions.

NOTE: This course content review is most effective as a continuous process that generates feedback throughout the year. ACT recommends, at minimum, monthly status update meetings for teachers and departments involved in the review.

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Geometry Course Standards	Gap 1	Gap 2	Comments
<b>A. Prerequisites</b>			
<b>1. Skills Acquired by Students in a Previous Course and Refined in This Course</b>			
a. Apply algebraic properties (e.g., commutative, associative, distributive, identity, inverse, substitution) to simplify algebraic expressions			
b. Solve single-step and multistep equations and inequalities in one variable			
c. Write linear equations in standard form and slope-intercept form when given two points, a point and the slope, or the graph of the equation			
d. Recognize the concept of slope as a rate of change and determine the slope when given the equation of a line in standard form or slope-intercept form, the graph of a line, two points, or a verbal description			
e. Graph a linear equation using a table of values, $x$ - and $y$ -intercepts, or slope-intercept form			
f. Find the probability of a simple event			
<b>B. Exploring the Skills and Strategies Underlying Mathematics</b>			
<b>1. Mathematical Processes Learned in the Context of Increasingly Complex Mathematical and Real-World Problems</b> (Note: These mathematical processes are the same for Algebra I, Geometry, Algebra II, and Precalculus.)			
a. Apply problem-solving skills (e.g., identifying irrelevant or missing information, making conjectures, extracting mathematical meaning, recognizing and performing multiple steps when needed, verifying results in the context of the problem) to the solution of real-world problems			
b. Use a variety of strategies to set up and solve increasingly complex problems			
c. Represent data, real-world situations, and solutions in increasingly complex contexts (e.g., expressions, formulas, tables, charts, graphs, relations, functions) and understand the relationships			
d. Use the language of mathematics to communicate increasingly complex ideas orally and in writing, using symbols and notations correctly			

Geometry Course Standards	Gap 1	Gap 2	Comments
e. Make appropriate use of estimation and mental mathematics in computations and to determine the reasonableness of solutions to increasingly complex problems			
f. Make mathematical connections among concepts, across disciplines, and in everyday experiences			
g. Demonstrate the appropriate role of technology (e.g., calculators, software programs) in mathematics (e.g., organize data, develop concepts, explore relationships, decrease time spent on computations after a skill has been established)			
h. Apply previously learned algebraic concepts in geometric contexts			

### C. Using Logic and Proof to Reason Mathematically

#### 1. Logic and Proof

a. Use definitions, basic postulates, and theorems about points, segments, lines, angles, and planes to write proofs and to solve problems			
b. Use inductive reasoning to make conjectures and deductive reasoning to arrive at valid conclusions			
c. Identify and write conditional and biconditional statements along with the converse, inverse, and contrapositive of a conditional statement; use these statements to form conclusions			
d. Use various methods to prove that two lines are parallel or perpendicular (e.g., using coordinates, angle measures)			
e. Read and write different types and formats of proofs including two-column, flowchart, paragraph, and indirect proofs			
f. Prove that two triangles are congruent by applying the SSS, SAS, ASA, AAS, and HL congruence statements			
g. Use the principle that corresponding parts of congruent triangles are congruent to solve problems			
h. Use several methods, including AA, SAS, and SSS, to prove that two triangles are similar, corresponding sides are proportional, and corresponding angles are congruent			



Geometry Course Standards	Gap 1	Gap 2	Comments
i. Use properties of special quadrilaterals in a proof			
<b>D. Identifying, Classifying, and Applying the Properties of Geometric Figures in Space</b>			
<b>1. Points, Lines, Planes, and Space</b>			
a. Identify and model plane figures, including collinear and noncollinear points, lines, segments, rays, and angles using appropriate mathematical symbols			
b. Identify vertical, adjacent, complementary, and supplementary angle pairs and use them to solve problems (e.g., solve equations, use in proofs)			
c. Identify corresponding, same-side interior, same-side exterior, alternate interior, and alternate exterior angle pairs formed by a pair of parallel lines and a transversal and use these special angle pairs to solve problems (e.g., solve equations, use in proofs)			
d. Use construction techniques, including straightedge and compass, to bisect and trisect segments and to create parallel and perpendicular lines, perpendicular bisectors, and angle bisectors			
e. Locate, describe, and draw a locus in a plane or space			
f. Apply properties and theorems of parallel and perpendicular lines to solve problems			
<b>2. Polygons</b>			
a. Identify and classify triangles by their sides and angles			
b. Identify medians, altitudes, perpendicular bisectors, and angle bisectors of triangles and use their properties to solve problems (e.g., find points of concurrency, segment lengths, or angle measures)			
c. Apply the Triangle Inequality Theorem to determine if a triangle exists and the order of sides and angles			
d. Solve problems involving the relationships formed when the altitude to the hypotenuse of a right triangle is drawn			

Geometry Course Standards	Gap 1	Gap 2	Comments
e. Apply the Pythagorean Theorem and its converse to triangles to solve mathematical and real-world problems (e.g., shadows and poles, ladders)			
f. Identify and use Pythagorean triples in right triangles to find lengths of the unknown side			
g. Identify and classify quadrilaterals, including parallelograms, rectangles, rhombi, squares, kites, trapezoids, and isosceles trapezoids, using their properties			
h. Identify and classify regular and nonregular polygons (e.g., pentagons, hexagons, heptagons, octagons, nonagons, decagons, dodecagons) based on the number of sides, the angle measures, and the side lengths			
i. Apply the Angle Sum Theorem for triangles and polygons to find interior and exterior angle measures given the number of sides, to find the number of sides given angle measures, and to solve real-world problems			
j. Apply the Isosceles Triangle Theorem and its converse to triangles to solve mathematical and real-world problems			
<b>3. Circles</b>			
a. Identify and define line segments associated with circles (e.g., radii, diameters, chords, secants, tangents)			
b. Determine the measure of central and inscribed angles and their intercepted arcs			
c. Find segment lengths, angle measures, and intercepted arc measures formed by chords, secants, and tangents intersecting inside and outside circles			
d. Solve problems using inscribed and circumscribed polygons			
<b>4. Solids</b>			
a. Identify and classify prisms, pyramids, cylinders, cones, and spheres and use their properties to solve problems			
b. Describe and draw cross sections of prisms, cylinders, pyramids, and cones			

Geometry Course Standards	Gap 1	Gap 2	Comments
<b>E. Comparing Congruent and Similar Geometric Figures</b>			
<b>1. Similarity and Congruence</b>			
a. Determine points or lines of symmetry and apply the properties of symmetry to figures			
b. Identify congruent figures and their corresponding parts			
c. Identify similar figures and use ratios and proportions to solve mathematical and real-world problems (e.g., finding the height of a tree using the shadow of the tree and the height and shadow of a person)			
d. Use the definition of similarity to establish the congruence of angles, proportionality of sides, and scale factor of two similar polygons			
e. Identify and draw images of transformations and use their properties to solve problems			
f. Apply relationships between perimeters of similar figures, areas of similar figures, and volumes of similar figures, in terms of scale factor, to solve mathematical and real-world problems			
g. Determine the geometric mean between two numbers and use it to solve problems (e.g., find the lengths of segments in right triangles)			
h. Identify and give properties of congruent or similar solids			
<b>F. Using Length, Area, Perimeter, and Volume to Find Quantities and Solve Problems</b>			
<b>1. Area and Perimeter</b>			
a. Find the perimeter and area of common plane figures, including triangles, quadrilaterals, regular polygons, and irregular figures, from given information using appropriate units of measurement			
b. Manipulate perimeter and area formulas to solve problems (e.g., finding missing lengths)			
c. Use area to solve problems involving geometric probability			

Geometry Course Standards	Gap 1	Gap 2	Comments
d. Find arc lengths and circumferences of circles from given information (e.g., radius, diameter, coordinates)			
e. Find the area of a circle and the area of a sector of a circle from given information (e.g., radius, diameter, coordinates)			
<b>2. Lateral Area, Surface Area, and Volume</b>			
a. Find the lateral area, surface area, and volume of prisms, cylinders, cones, and pyramids in mathematical and real-world settings			
b. Use cross sections of prisms, cylinders, pyramids, and cones to solve volume problems			
c. Find the surface area and volume of a sphere in mathematical and real-world settings			
<b>G. Relating Geometric Ideas to the Coordinate Plane</b>			
<b>1. Coordinate Geometry</b>			
a. Use slope to distinguish between and write equations for parallel and perpendicular lines			
b. Apply the midpoint and distance formulas to points and segments to find midpoints, distances, and missing information			
c. Use coordinate geometry to solve problems about geometric figures (e.g., segments, triangles, quadrilaterals)			
d. Write equations for circles in standard form and solve problems using equations and graphs			
e. Determine the effect of reflections, rotations, translations, and dilations and their compositions on the coordinate plane			

Geometry Course Standards	Gap 1	Gap 2	Comments
<b>H. Investigating and Applying Basic Ideas of Trigonometry</b>			
<b>1. Introduction to Trigonometry</b>			
a. Apply properties of $45^\circ$ - $45^\circ$ - $90^\circ$ and $30^\circ$ - $60^\circ$ - $90^\circ$ triangles to determine lengths of sides of triangles			
b. Find the sine, cosine, and tangent ratios of acute angles given the side lengths of right triangles			
c. Use trigonometric ratios to find the sides or angles of right triangles and to solve real-world problems (e.g., use angles of elevation and depression to find missing measures)			